3維非定常浮體表面邊界條件

浮體在水面或水中受波運動，勢函數為ϕ，作自由浮體運動、或被張緊繫留索繫留，靜止時重心位置在$(\bar{x}_0, \bar{y}_0, \bar{z}_0)$，浮心位置在$(x_b, y_b, z_b)$，達平衡狀態時，移至$(x_o, y_0, z_o)$，浮體運動的水平、垂直及回轉角振幅為$\delta_1$、$\delta_2$及$\delta_3$(以逆時針方向為正)，兩者間關係為

$$\bar{x} = x - (x_0 - \bar{x}_0) + \delta_3 (z - \bar{z}_0) - \delta_2 (y - \bar{y}_0)$$

$$\bar{y} = y - (y_0 - \bar{y}_0) + \delta_2 (x - \bar{x}_0) - \delta_1 (z - \bar{z}_0)$$

$$\bar{z} = z - (z_0 - \bar{z}_0) + \delta_1 (y - \bar{y}_0) - \delta_3 (x - \bar{x}_0)$$

若$z = \varsigma(x, y; t)$表示浮體水中部分表面方程式，則由上式得

$$\begin{aligned} \zeta = & -\left[(x_0 - \bar{x}_0) + \delta_2 (y - \bar{y}_0) - \delta_3 (z - \bar{z}_0) \right] \zeta_x \\ & -\left[(y_0 - \bar{y}_0) + \delta_1 (z - \bar{z}_0) - \delta_2 (x - \bar{x}_0) \right] \zeta_y \\ & +\left[(z_0 - \bar{z}_0) + \delta_3 (x - \bar{x}_0) - \delta_1 (y - \bar{y}_0) \right] \end{aligned}$$

上式右邊ζ_x，ζ_y及-1為靜止狀態時，浮體水中部份表面S法線在x ,y, z方向分量。由於

$$d\zeta / dt = u\zeta_x + v\zeta_y + w\zeta_z + \zeta_t = 0$$

若流體運動具速度勢$\Phi(x, y, z; t)$，則上式可寫成

$$\Phi_x \zeta_x + \Phi_y \zeta_y - \Phi_z + \zeta_t = 0$$

浮體表面，浮體運動與水粒子運動必須連續得

$$\begin{aligned} \frac{\partial \Phi}{\partial \upsilon} = & \frac{\partial x_0}{\partial t}\frac{\partial x}{\partial \upsilon} + \frac{\partial y_0}{\partial t}\frac{\partial y}{\partial \upsilon} + \frac{\partial z_0}{\partial t}\frac{\partial z}{\partial \upsilon} \\ & + \frac{\partial \delta_1}{\partial t}\left[(z - \bar{z}_0)\frac{\partial y}{\partial \upsilon} - (y - y_0)\frac{\partial z}{\partial \upsilon} \right] \\ & + \frac{\partial \delta_2}{\partial t}\left[(x - \bar{x}_0)\frac{\partial z}{\partial \upsilon} - (z - \bar{z}_0)\frac{\partial x}{\partial \upsilon} \right] \\ & + \frac{\partial \delta_3}{\partial t}\left[(y - \bar{y}_0)\frac{\partial x}{\partial \upsilon} - (x - \bar{x}_0)\frac{\partial y}{\partial \upsilon} \right] \end{aligned}$$

浮體質量為m，通過重心與x ,y ,z軸平行慣性力距以I_x, I_y, I_z表示。作用於浮體壓力除重力、流體壓力p 外，尚有復原力R，其在x、y、z方向分力以R_x, R_y, R_z表示。復原力所引起對重心力距若以N表示，其對x、y及z軸分力距以M_x, M_y, M_z表示，則浮體第1近似運動方程式為

$$
\left.\begin{aligned}
m\frac{d^2x_0}{dt^2} &= \iint_A p\frac{\partial x}{\partial \upsilon}dA + R_x \\
m\frac{d^2y_0}{dt^2} &= \iint_A p\frac{\partial y}{\partial \upsilon}dA + R_y \\
m\frac{d^2z_0}{dt^2} &= \iint_A p\frac{\partial z}{\partial \upsilon}dA + R_z
\end{aligned}\right]
$$

$$
\left.\begin{aligned}
I_x\frac{d^2\delta_1}{dt^2} &= \iint_A p\left[\frac{\partial z}{\partial \upsilon}(y-\bar{y}_0) - \frac{\partial y}{\partial \upsilon}(z-\bar{z}_0)\right]dA + M_x \\
I_y\frac{d^2\delta_2}{dt^2} &= \iint_A p\left[\frac{\partial x}{\partial \upsilon}(z-\bar{z}_0) - \frac{\partial z}{\partial \upsilon}(x-\bar{x}_0)\right]dA + M_y \\
I_z\frac{d^2\delta_3}{dt^2} &= \iint_A p\left[\frac{\partial y}{\partial \upsilon}(x-\bar{x}_0) - \frac{\partial x}{\partial \upsilon}(y-\bar{y}_0)\right]dA + M_z
\end{aligned}\right]
$$

$$
\begin{aligned}
R_x &= -\iint_S \rho g z_0 \frac{\partial x}{\partial \upsilon}dA \\
R_y &= -\iint_S \rho g z_0 \frac{\partial y}{\partial \upsilon}dA \\
R_z &= -\iint_S \rho g z_0 \frac{\partial z}{\partial \upsilon}dA
\end{aligned}
$$

$$
\left.\begin{aligned}
M_x &= -\iint_A \rho g \delta_1 (y-\bar{y}_0)\left[\frac{\partial z}{\partial \upsilon}(y-\bar{y}_0) - \frac{\partial y}{\partial \upsilon}(z-\bar{z}_0)\right]dA \\
M_y &= -\iint_A \rho g \delta_2 (x-\bar{x}_0)\left[\frac{\partial x}{\partial \upsilon}(z-\bar{z}_0) - \frac{\partial z}{\partial \upsilon}(x-\bar{x}_0)\right]dA \\
M_z &= -\iint_A \rho g \delta_3 (z-\bar{z}_0)\left[\frac{\partial y}{\partial \upsilon}(x-\bar{x}_0) - \frac{\partial x}{\partial \upsilon}(y-\bar{y}_0)\right]dA
\end{aligned}\right]
$$