NUMERICAL STUDY ON THE BREAKING OF SOLITARY WAVE ON SLOPES

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ABSTRACT

Numerical study of the breaking criterion of solitary waves propagating on slopes was carried out by means of boundary element method, the algorithm was based on the Lagrangian description and finite differencing to time. The shoaling and breaking processes of solitary waves on various kinds of slopes are studied. According to the criterion defined as the horizontal velocity of water particle on wave crest equals to the wave celerity, our suggestions of breaking indices for slopes 1:10 to 1:50 are laid out, with which an empirical formula for the breaking indices was presented. In this article, the deformation of the wave profiles as well as the distribution of fluid velocities at the breaking region for slopes 1:30 to 1:50 are shown. Our results showed that when slope varies from 1:10 to 1:25, the breaker type was classified by the angle of wave profile on the wave crest at breaking, it was considered as plunging breakers if the angle is smaller than $\theta = 90^0$, otherwise they are classified as surging breaker. Furthermore, when the slope varies from 1:30 to 1:50, the breakers are mostly classified as spilling breaker.

1 INTRODUCTION

Numerical studies for shoaling of solitary waves are developed by numerous researchers. Prior discussions in detail for the propagation of solitary wave are studied by Madsen and Mei [1], they simulated the solitary waves passing through a mild slope and onto a shelf, reasonable agreement compared with experimental data was obtained. The propagation of solitary wave and its run-up against vertical wall are studied by Nakayama [2] using the boundary element method. With the mixed Eulerian-Lagrangian technique, Kioka [3] investigated the deformation and velocity field of shallow-water breaking waves. The resultant of two types of breaker, plunging and spilling breakers, are shown. Kim *et al.* [4] discussed the generation, propagation and run-up of solitary waves by using the boundary integral equation solution, including one single solitary wave and two successive solitary waves. Grilli *et al.* [5][6] used a fully nonlinear potential flow wave model to explore the shoaling and breaking of solitary waves on slopes, with their further extension, a breaking criterion was then presented. Based on the Lagrangian description and finite differencing of time step, the generation, propagation and deformation of solitary waves are simulated numerically by Chou and Shih [7] with boundary element method, time histories of a soliton running up on a sloping beach and onto a shelf as well as over a submerged obstacle are then presented. Further studies of the phenomena for solitary waves on various slopes including shoaling and wave breaking are then investigated by Chou and Ouyang [8][9], the breaking indices for slopes s=1:1 to 1:25 are proposed.

This article is a further extension of the previous studies by Chou, the shoaling and breaking of solitary waves on slopes s=1:30 to 1:50 are investigated. Based on the Lagrangian description and finite differencing of time step, a fully nonlinear boundary condition on free water surface was substituted, an algorithm to generated waves with piston type is also implanted in the method. As solitary wave propagates up to the sloping beach, the breaking criterion is defined as the equalization between the water particle's horizontal velocity on the crest and the wave celerity. The breaking indices are laid out and the classification of breaking type are discussed, the deformation of wave profile and distribution of fluid velocities are presented as well.

2 THEORETICAL ANALYSIS

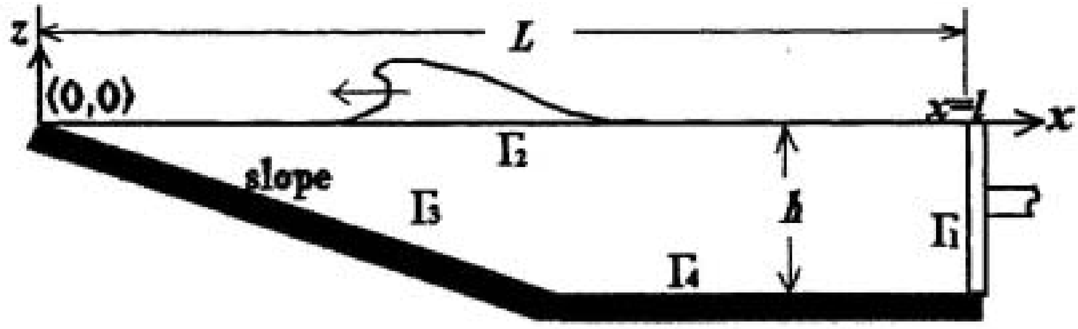

Figure 1: Definition sketch of the numerical flume.

By substituting a fully nonlinear boundary condition on the free water surface, the two-dimensional boundary element method was used to simulate the breaking of solitary waves on mild slopes. The governing equation, boundary conditions and numerical methods are briefly reviewed here, all details can be referred to Chou *et al.* [8][9] and Shih [7]. As shown in Figure 1, the fluid within the region is assumed to be inviscid and incompressible, and the flow is irrotational. According to Green's Second Identity, when the velocity potential satisfies the Laplace equation.and it's second derivative exists, the velocity potential can be obtained by the velocity potential on the boundary, $\Phi(\xi,\eta;t)$, and it's normal derivative, $\partial\Phi(\xi,\eta;t)/\partial n$, i.e.

$$c\Phi(x,z;t)=\frac{1}{2\pi}\int_{\Gamma}\left[\frac{\partial\Phi(\xi,\eta;t)}{\partial n}\ln\frac{1}{r}-\Phi(\xi,\eta;t)\frac{\partial}{\partial n}\ln\frac{1}{r}\right]ds \tag{1}$$

$$c=\begin{cases}1 & \text{inside the fluid domain}\\ 1/2 & \text{on the smooth boundary}\\ 0 & \text{outside the fluid domain}\end{cases}$$

where $r=\left[(\xi-x)^2+(\eta-z)^2\right]^{1/2}$.

The boundary conditions on the pseudo wave generator are assumed that the fluid velocity normal to the paddle is the same as the horizontal moving velocity of the piston-type wave generator. On free water surface, fully nonlinear boundary condition was applied, with the assumption that the atmospheric pressure is constant and equals to zero. For impermeable slope and seabed, the velocities normal to the boundaries are null. Linear elements are adopted to solve eqn(1), forwarddifference in time was used to obtain the position of free water surface at the following time. The double nodes numerical technique (Chou, 1988) was applied to the corners which have two different boundary conditions. Numerical accuracy are verified by the conservation of mass and energy.

3 NUMERICAL RESULTS AND DISCUSSIONS

3.1 The deformations of wave profiles

To determine whether or not the waves break, various kinds of breaking criterion have been suggested, e.g., the horizontal velocities of fluid particles on the crest reached the wave celerity, the angle of wave profile on the crest reaches $\theta = 120^0$, the front face of wave profile appears vertical and the related wave height reaches a specific value. The validity of breaking criterion for solitary waves on slopes have been discussed in detail by Chou and Ouyang [8][9], which waves are regarded as critical breaking when the ratio of horizontal velocities of water particles on wave crest to wave celerity equals one, yet, for some cases which couldn't reach one, the wave front already became vertical are also considered as critical breaking. These criteria are adopted for discussion. All results in this paper can reach $u/C = 1$, where u is the average horizontal velocities of water particles near the crest and C is the wave celerity calculated by $C = \sqrt{g(H+h)}$. Deformations of wave profiles for solitary waves shoaling on slopes s=1:30 to 1:50 are shown in Figure 2 to Figure 6, with the angles of wave profiles marked on both the top of the crest when $u/C = 1$ and at breaking.

Figure 2 shows the results of waves on slope s=1:30, where Figure 2(a) illustrates the case with incident wave height of $H_0/h_0 = 0.1$ approaching the shoreline, the angle of the breaking wave profile is about $\theta = 117^0$ as the wave reaches $u/C = 1$. For $H_0/h_0 = 0.2$ shown in Figure 2(b), the wave breaks presently after $u/C = 1$ was reached, the angle of which is $\theta = 104^0$ at $u/C = 1$, and $\theta = 100^0$ when the wave breaks. Figure 2(c) and Figure 2(d) are the cases with $H_0/h_0 = 0.3$ and 0.4, the angles at the criterion are 101^0 and 115^0 respectively.

Figure 3 ~ Figure 6 represents the deformations of wave profiles for waves on slope s=1:35~1:50. The front face of the wave profile becomes steeper until they break, but it maintained a bit symmetrical at breaking. By observing the angles marked on the top of the crest and listed in Table 1, we found an ambit that most angles could be bounded, say, $\theta = 90^0$ to $\theta = 120^0$. This features are quite obvious when $H_0/h_0 = 0.2$ ~ 0.4, e.g., the angles at breaking within slope s=1:30~1:50 for incident wave height of $H_0/h_0 = 0.4$ are 103^0, 99^0, 99^0, 101^0 and 93^0 respectively. However, some results of cases with $H_0/h_0 = 0.1$ may obtained a larger angle, by observing the ratio of x_b/h_0 in Table 1, we reasoned that might due to the circumstance that the simulated waves approached extremely close to the shoreline, as a result of the capable achievement of our simulations so far for cases with smaller incident wave heights on mild slopes.

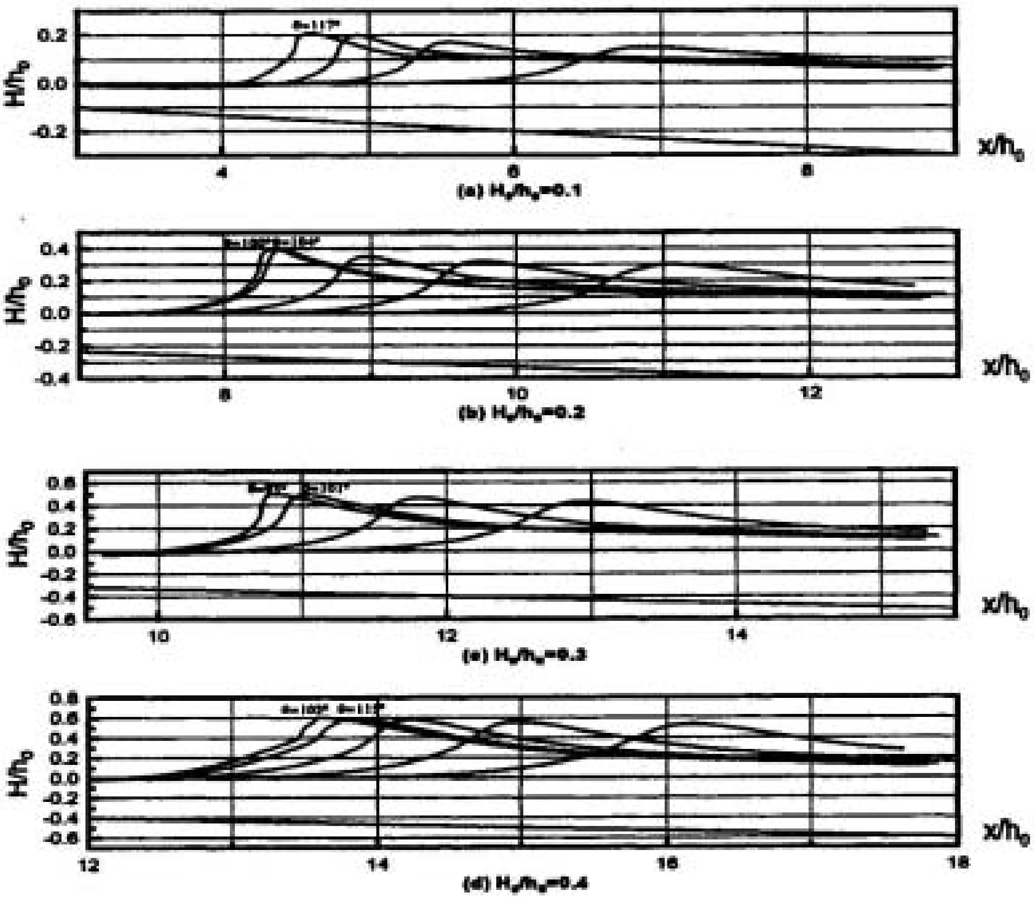

Figure 2: The deformation of solitary waves on slope s=1:30

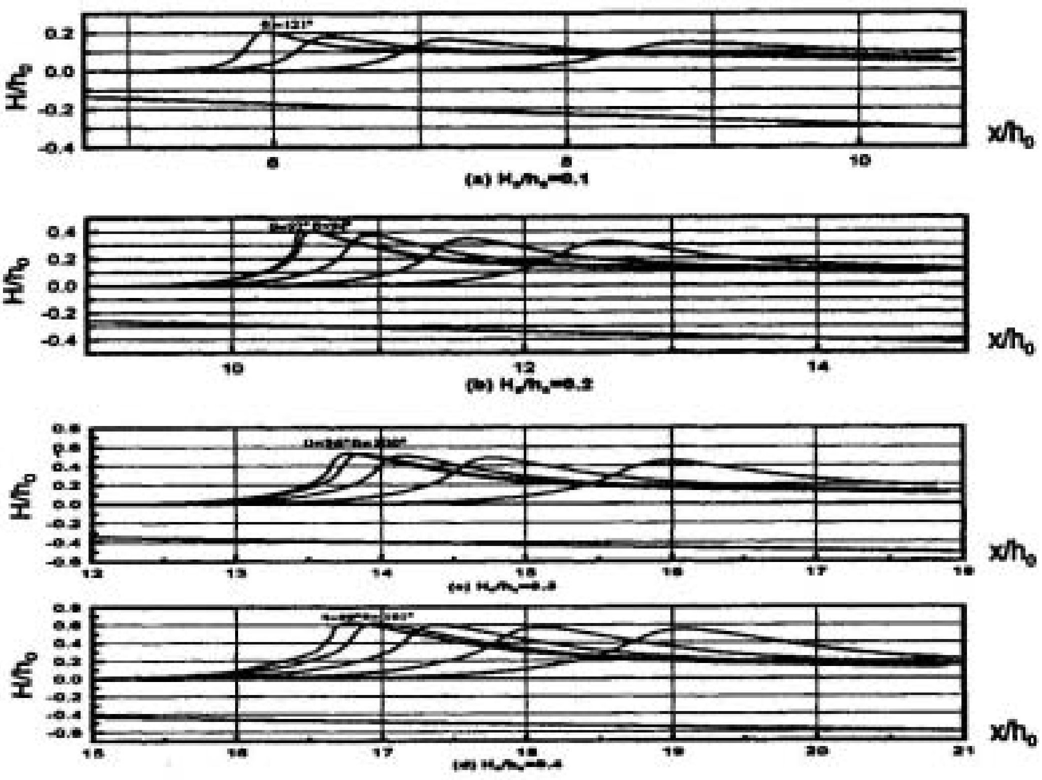

Figure 3: The deformation of solitary waves on slope s=1:35.

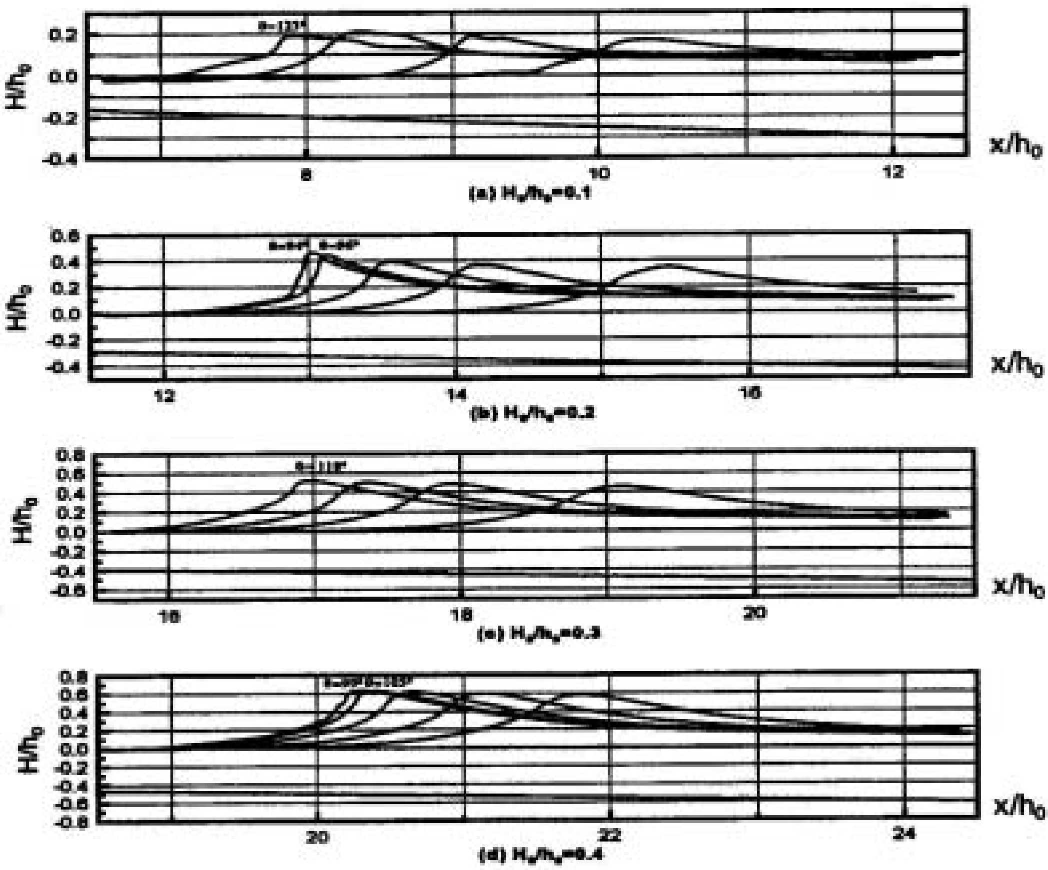

Figure 4: The deformation of solitary waves on slope s=1:40.

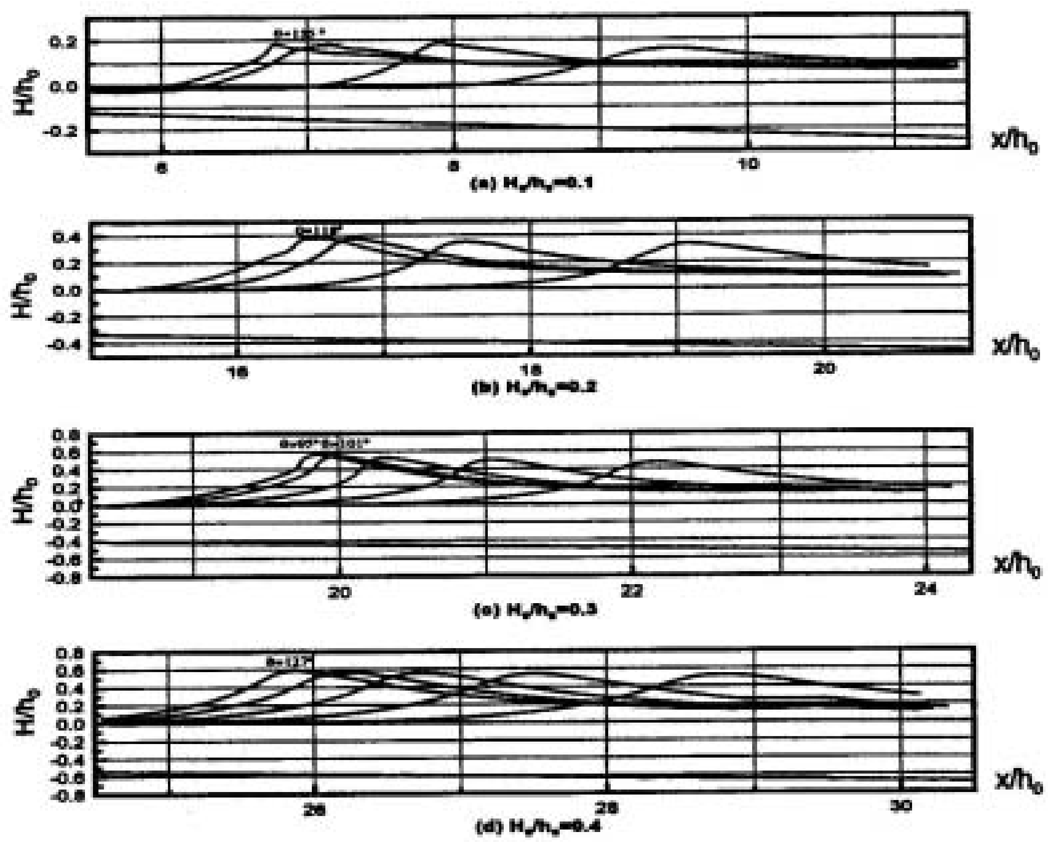

Figure 5: The deformation of solitary waves on slope s=1:45.

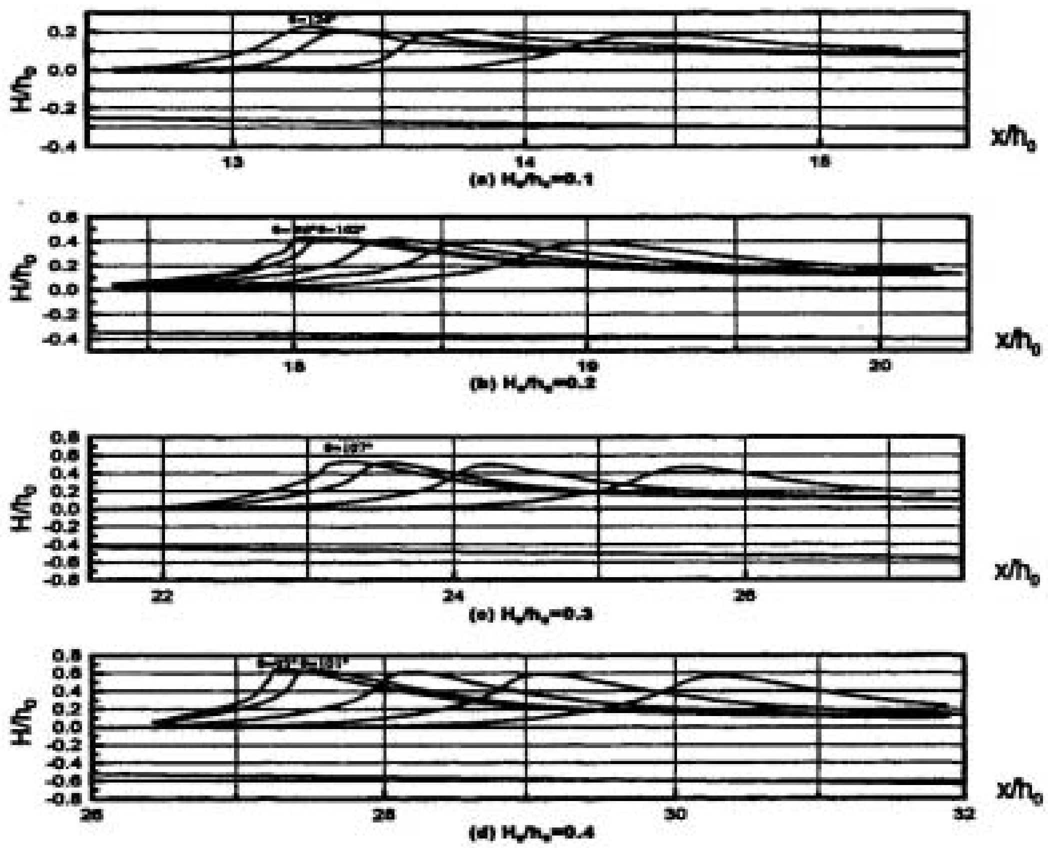

Figure 6: The deformation of solitary waves on slope s=1:50.

3.2 The distribution of fluid velocities

The distribution of fluid velocities at breaking for waves on slope s=1:30 are showned in Figure 7, with the comparison between incident waves heigth of H_0/h_0 =0.1~0.4, in Figure 7(d), a greater velocity appeared on the crest evidently. Also, in Figure 7(b)&(c), the greater velocity appeared on both the crest and the upper wave front. Figure 8 shows the distribution of fluid velocities at breaking for waves on slope s=1:35 with different incident wave heights. The movement of fluid particles speed up evidently near the crest. According to Chou and Ouyang [9], plunging breakers existed a strong velocity occurred virtually on the whole wave front with a relatively non-uniform distribution of fluid velocities. Thus, phenomena for waves simulated here are quite different to that of plunging breakers. Figure 9~11 shows the cases of waves breaking on slopes s=1:40 to 1:50 respectively, the resemblance between the distribution of the greater wave velocity occurred on the crest are similar to that in Figure 8. By combining the distribution of fluid velocities with the features of the angle on the crest, also refer our numerical results to experimental observations presented by previous scholars, these waves are classified as spilling breakers.

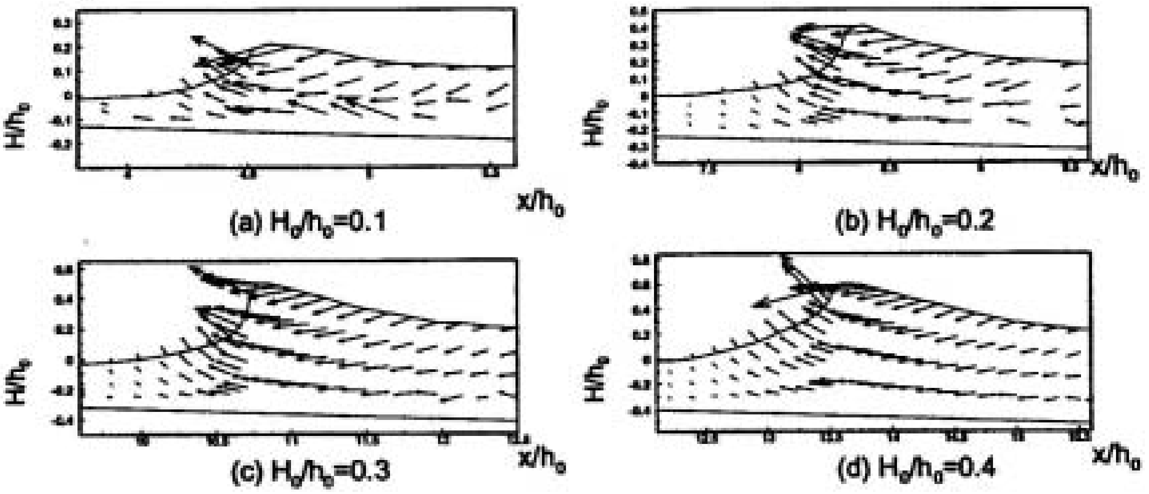

Figure 7: The distribution of fluid velocities of wave breaking on slope s=1:30.

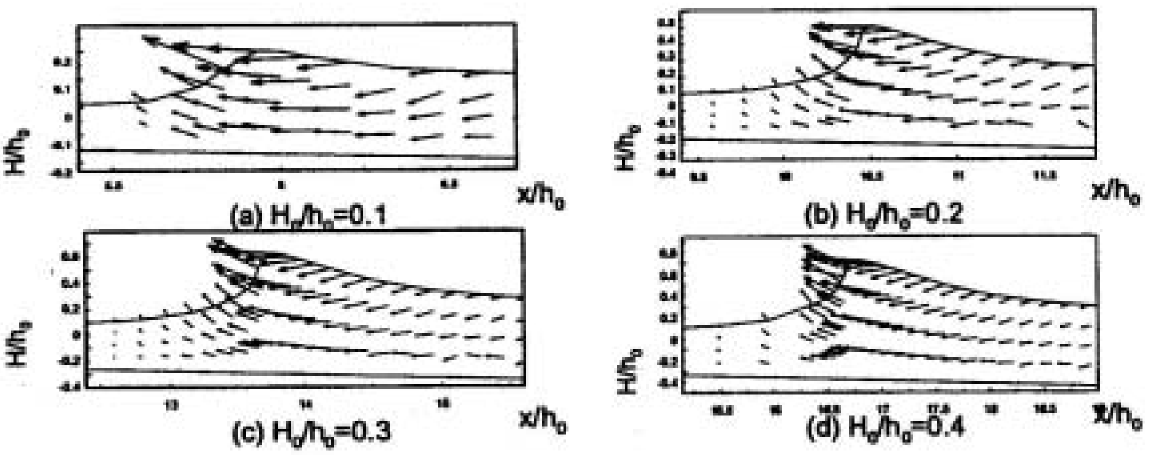

Figure 8: The distribution of fluid velocities of wave breaking on slope s=1:35.

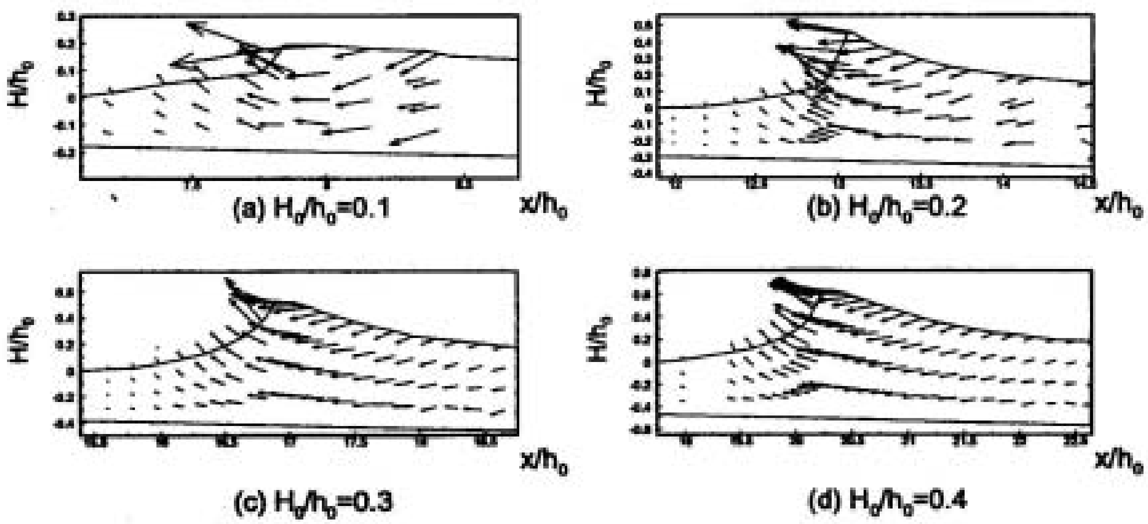

Figure 9: The distribution of fluid velocities of wave breaking on slope s=1:40.

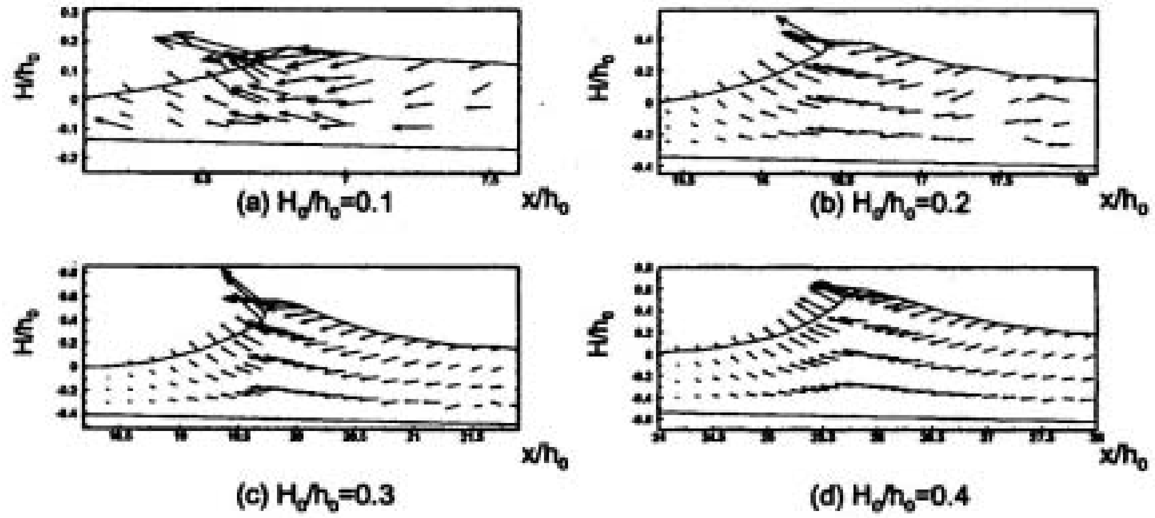

Figure 10: The distribution of fluid velocities of wave breaking on slope s=1:45.

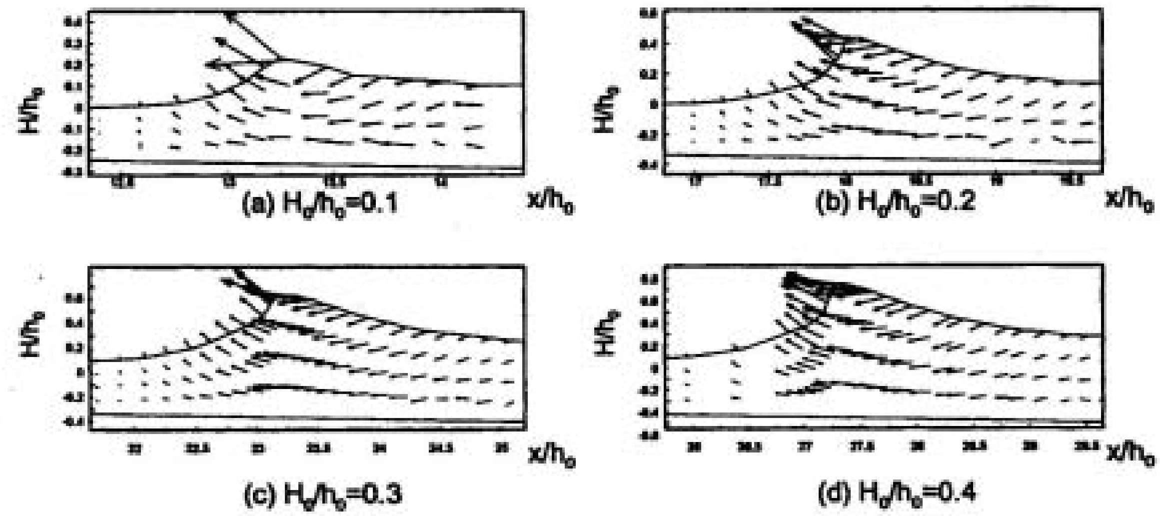

Figure 11: The distribution of fluid velocities of wave breaking on slope s=1:50.

3.3 The breaking indices

As mentioned above, the condition of $u/C=1$ was adopted as a criterion of wave breaking. According to this, the breaking indices for solitary waves on slopes s=1:30 to 1:50 are listed in Table 1, in which the subscriptions b denotes the values at the critical breaking. The related breaking wave height, H_b/h_b, are over one except for waves of $H_0/h_0=0.1$ on slopes s=1:40 and 1:50. Considering the breaking wave height H_b/h_0 for $H_0/h_0=0.2$, the values revealed closely, same circumstances are found when $H_0/h_0=0.3$ and 0.4, that is to say, the variation of slope does not affect evidently to the breaking wave height as the slopes are mild.

Table 1. The breaking indices for solitary waves on slopes s=1:30 to 1:50.

slope	H_0/h_0	H_b/h_0	H_b/h_b	h_b/h_0	x_b/h_0	θ	θ at breaking
1:30	0.1	0.21	1.36	0.15	4.59	117	117
	0.2	0.40	1.42	0.28	8.38	104	100
	0.3	0.50	1.39	0.36	10.82	101	93
	0.4	0.60	1.28	0.46	13.88	115	103
1:35	0.1	0.21	1.21	0.17	5.93	121	121
	0.2	0.40	1.34	0.30	10.55	94	92
	0.3	0.53	1.33	0.40	13.86	100	96
	0.4	0.62	1.27	0.48	16.95	101	99
1:40	0.1	0.19	0.96	0.20	7.9	123	123
	0.2	0.45	1.36	0.33	13.11	96	94
	0.3	0.52	1.22	0.42	16.97	118	118
	0.4	0.63	1.23	0.51	20.40	105	99
1:45	0.1	0.18	1.19	0.15	6.76	135	135
	0.2	0.39	1.04	0.37	16.58	118	118
	0.3	0.56	1.26	0.44	19.97	101	95
	0.4	0.63	1.19	0.53	23.92	103	101
1:50	0.1	0.23	0.87	0.27	13.25	126	126
	0.2	0.43	1.18	0.36	18.16	102	96
	0.3	0.53	1.15	0.47	23.28	107	107
	0.4	0.64	1.16	0.55	27.53	101	93

3 Conclusions

The shoaling and breaking of solitary waves propagating on mild slopes with s=1:30~1:50 are investigated in this study. The deformation of wave profiles and the distribution of fluid velocities at the breaking region are then presented. All simulated results in this paper can reach $u/C=1$. An ambit around 90^0 to 120^0 was observed which the angles of the wave profiles could be bounded when breaking, although the wave profiles became steeper when shoaling, the wave profiles maintained a bit symmetrical as it breaks. The distribution of fluid velocities shows that a greater velocity occurred on the crest evidently. Conclude these observations, also refer our numerical results with experimental observations presented by previous scholars, waves simulated here are classified as spilling breakers.

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