Experimental Determination on the Reflectance Characteristics of Undulating Submerged Obstacle: Comparison Between Regular and Irregular Wave Response

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ABSTRACT

In this study, physical experiments are conducted with various combination of undulating breakwaters collocated with various wave conditions, including regular and irregular waves. The wave reflectivity of various combinations of sinusoidal breakwaters is discussed by the variation of reflectance; the optimizations of variety of sinusoidal breakwater are preliminary analyzed. The interaction of surface waves scattering by an undulating seabed such as sinusoidal bars is one of the fundamental importance relates to sediment transportation, however, such steep and high sediment bed shaped may be instable in the prototype. The characteristics of stabilized multi-array impermeable sinusoidal submerged breakwater are investigated experimentally in this article, and the interactions between wave and the obstacle are discussed. This study discusses merely on the properties of the reflection coefficient of the sinusoidal breakwaters and varying combinations of the breakwaters. The optimization of the disposition of sinusoidal breakwaters are also discussed.

KEY WORDS: Sinusoidal breakwater; series submerged breakwater; reflection coefficient; wave attenuation; irregular wave.

INTRODUCTION

Quite recently, the increasing emphasis on the hydrophilic facilities is greatly interested in view of the preserving of natural landscape. The enforcement of a so-called "amenity-oriented policy" was subsequently established to promote the comprehensive development of port and harbor facilities, and many constructions are substituted by ecological engineering method. Offshore structures, such as breakwaters and seawalls, have been designed and constructed to produce calmness and ensure urban safety by reducing huge wave force. Many submerged breakwaters in relation to "Sub dike", including square shapes, trapezoid and triangular forms impermeable protecting embankments, are being extensively investigated and discussed. In recent years, many scholars have studied a variety of offshore-submerged breakwater, including changing the external form or shape of submerged obstacles, the properties of permeability, the quantity of submerged obstacles, and the interval between each obstacle, and variation in wave conditions. Based on Miles' (1981) theory, Hsu et al. (2003) used an evolution equation for mild-slope equation to study the Bragg reflection of water waves over multiple-composite submerged breakwaters. Relative to the subject, they investigated numerically the Bragg scattering of water waves by multiple composite artificial bars based on the hyperbolic equation of Zhang et al. (1999). Their results exhibited the performance of the Bragg resonance for multiple composite artificial bars are greatly improved by increasing both the relative bar height and the quantity of bars at various intervals.

The scattering and trapping of water waves using 3-D submerged topography was first considered by Porter and Porter (2001). The interaction of linear surface gravity waves with 3-D periodic topography was concerned, considering that the scattering by the topography of parallel-crested obliquely incident waves and the propagation of trapping modes along the periodic topography are formulated and numerically solved. Porter and Porter (2003) also investigated the interactions between surface water waves and periodic bed forms in three different situations, including the scattering of given incident waves by finitely many periods of topography, and concern respectively the existence of unforced waves over periodically varying beds of an infinite and finite extent. They obtained the exact solution for ripple bed scattering based on full linear theory, with which they have compared previous approximations derived by using the modified mildslope equation. The transfer matrix formulation can be applied to scattering by any topography, without incurring errors such as those that arise from bed discretization. In particular, the approach could be used to examine scattering by random periodic beds.

Komarova and Newell (2000) accurately estimated the characteristic features of sand banks by deriving nonlinear amplitude equations governing the couple dynamics of sand waves and sand banks. Their results are consistent with physical measurements. Yu and Mei (2000) derived a quantitative theory to describe the formation mechanism and evolution of sand bars by assuming that the slopes (of bars and waves) are comparably gentle, and that sediment motion is dominated by the bedload, which is unlike that for sand ripples governed by instability. Ardhuin and Magne (2007) presented a theory that describes the scattering of random surface gravity waves by small bottom amplitude topography with an irrotational uniform current. Their results reveals that the application of this theory to waves over current-generated sand

waves suggests that forward scattering can be significant, whereas backscattering is generally weaker.

Several related investigations have been carried out theoretically and experimentally for the predictions and discussion of the propagation of water waves over undulation seabed topography, and are mostly focus on the Bragg resonance problems which occurred between surface waves and undulated bottom. The Bragg resonance involves two monochromatic surfaces and respectively one and two bottom components, and was investigated by Liu and Yue (1998) by studying the generalized Bragg scattering of surface waves over a wavy bottom. They proceeded a step further on the investigation of higher order (quartet) Bragg resonance (Oblique sub- and super- harmonic Bragg resonance, see also Alam et al. 2010) involving two incident wave components and a bottom ripple component. Elgar et al. (2003) used field experiments to discussed the Bragg resonance of ocean wave from sandbars, and observed the resonance for a range of incident wave conditions. Alam et al. (2009a, 2009b) developed both analytical and numerical method to study the mechanism of Bragg resonance and the interaction of surface/interfacial waves over a rippled bottom caused by the resonant interaction among surface waves and bottom undulations of waves in a two-layer fluid. Yu and Howard (2010) developed an exact theory for linear irrotational motion over a corrugated bottom to study the higher-order Bragg resonances of water waves occurring when the corrugation wavelength is close to an inter-multiple of half a water wavelength. The primary Bragg resonance was first examined with frequencies close to the first-order reflections by the authors (Yu and Howard, 2007). Linton (2011) established the existence of a band-gap structure with waves propagating over periodic arrays of submerged horizontal circular cylinders in deep water. The waves were traveling at an oblique angle as well as normal to the cylinder axes. Their results indicated that the evanescent interactions between the cylinders play a crucial role in their creation, and the heuristic arguments that often posit to that explain Bragg resonance is inadequate in this case.

This article discusses the properties of the reflection coefficient of the sinusoidal breakwaters and various combinations of the submerged obstacles. The diversity in the reflection coefficient cause by varying height of the breakwater under various wave conditions, wave heights, and wave periods are discussed individually by regular wave and irregular wave. The diversity of reflection coefficients was mitigated by varied breakwater widths, and the reflection coefficient increased along with the breakwater's width and height.



Fig.1 Schematic layout of wave flume and experimental setup

EXPERIMENTAL SETUP

Hydraulic model tests were carried out in a 16m wave flume (is now enlarged to 21 m) located at the Fluid Mechanics Lab of Tungnan University. As shown schematically in Fig. 1, the channel is 80 cm wide and 60 cm high, and the still water level is 20 cm. The heights D of the embankment are 5 cm, 10 cm, 15 cm, and 20 cm, respectively. The

embankment widths *w* are 10 cm, 20 cm, and 30 cm, and the wave periods *T* range from 0.5 to 1.5 sec (i.e., $\sigma^2 h / g = 3.2$ to 0.36). The wave height H = 1.4 cm. Because the wave flume is relatively short, the time for wave generation is 45-60 seconds (depending on the wave period), and the wave gauge sampling time is 20Hz. To reduce the influence of the reflected waves, the wave flume is presently enlarged to 21m to make it more suitable for future experimental research.

A piston-type wave generator was located at one end of the flume with an absorbing 1:2.5 slope at the other end. The wave generator system was control by a Wave Generator Filter Unit produced by the Canadian Hydraulics Center. The experiments were carried out both in regular and irregular waves to confirm the efficacy of the designed breakwater. Waves were measured using five capacitance wave gauges with an adapter linked to a PC (Personal Computer). The incident wave heights were measured with the first gauge whereas the 2nd and 3rd gauges were used for the calculation of reflection coefficient, and the 4th and 5th gauges were used for transmission wave measurement. The model was installed in the middle of the tank with a distance of 10 m from the generator. The sinusoidal breakwaters were individually prefabricated in concrete in small pieces and were joined to form a row (Fig. 2), the tests were carried out with 60 combinations of sinusoidal breakwaters collocated with various wave conditions. The concrete specimens remained stable because of their own body weight, and were not moved by the impact of wave actions. However, higher submerged breakwater, (e.g. D=20 cm submerged breakwater) required to be fixed by steel bar to avoid swinging under wave forces.



Fig.2 Illustration of sinusoidal obstacle

A variety of different shape-steepness D/w (similar to wave steepness, H/L), and various incident wave conditions of wave periods $T(T_{1/3})$ and wave high $H(H_{1/3})$, the variation of reflection coefficient K_r was obtained under the conditions of variety with various D/w. The deformation of the waveform and wave-breaking effect were researched simultaneously by a CCD camera (digital video camera), and verify the measuring results via wave gauges.

The transmission of surface waves is a disturbance oscillating motion similar to a sinusoidal function. The trajectory of a water particle rotates elliptically within the flow field, and accomplishes the formation of similar wavelike dunes on the terrain. Thus, the goal here is to investigate the interactions and relations between such special terrain and incoming waves. This study investigates the relationship between the wave reflectivity and sharpness of the contour of breakwater by a series of SIN-shaped submerged breakwaters.

EXPRESSION OF WAVE REFLECTANCE

Random Wave Generation

The Bretschneider-Mitsuyasu spectrum is used as the target spectrum for the generation of irregular waves, which can be expressed as:

$$S_0(f) = 0.257 H_{1/3}^2 T_{1/3}^{-4} f^{-5} \exp[-1.03(T_{1/3}f)^{-4}]$$
(1)

where $H_{1/3}$ is the significant wave height, $T_{1/3}$ is the significant wave period, and *f* is the frequency. According to Goda (2000), the spectral peak frequency, f_p , is related to the significant wave period, T_p :

$$f_p = 1/T_p \tag{2}$$

where $T_p = 1.05 T_{1/3}$. Similar to that in physical applications of the wave generation theory, a transfer function must be multiplied to obtain the spectrum for the motion of the numerical wave generator, thus:

$$S(f) = \alpha(f)^2 \cdot S_0(f) \tag{3}$$

 $\alpha(f)$ denotes the transfer function of wave amplitude and the stroke of wave paddle, whereas in the present study, a piston type wave generator was investigated, hence,

$$\alpha(f) = \frac{\sinh kh \cosh kh + kh}{2\sin^2 kh} \tag{4}$$

k is the wave number, h is the water depth. Neglect the period which are excessively small or large, i.e. the associate period resolution can be express as:

$$T_{\min} < T < T_{\max}$$

where $T_{\min} = 0.5$ sec and $T_{\max} = 4.5$ sec was chosen presently. The surface fluctuations can be expressed by:

$$\zeta(t) = \sum_{n=1}^{N} \sqrt{2dfS(f_n)} \cdot \cos(\sigma_n t - \varepsilon_n)$$

$$\sigma_n = 2\pi f_n$$
(6)
(7)

here ε_n and N denotes random variable number between $0 \sim 2\pi$ and total number of sampling, respectively.

Expression of Wave Reflectance

The wave attenuation of the sinusoidal submerged obstacle can be estimate by the reflection and transmission of wave energy, the reflection coefficient are estimated by the method of Goda and Suzuki (1976) based on the time histories of water elevations, which are measured by five wave gages on the free water surface, the amplitude are analyzed by the FFT (Fast Fourier Transform) technique, thus, the reflection coefficients K_r were estimated as follows:

The composite wave profiles of incident and reflected waves at location $x=x_1$ and $x=x_1 + \Delta l$ can be expressed as:

$$\eta_{\rm l} = (\eta_{\rm i} + \eta_{\rm r})_{x=x1} = A_{\rm l} \cos \sigma t + B_{\rm l} \sin \sigma t \tag{8}$$

$$\int A_{\rm l} = a_{\rm i} \cos\theta_{\rm i} + a_{\rm r} \cos\theta_{\rm r} \tag{0}$$

$$B_{l} = a_{l} \sin \theta_{l} + a_{r} \sin \theta_{r}$$
(9)

$$\eta_2 = (\eta_1 + \eta_r)_{r=r_1 + \Lambda t} = A_2 \cos \sigma t + B_2 \sin \sigma t \tag{10}$$

$$\begin{cases} A_2 = a_i \cos(\theta_i + k\Delta l) + a_r \cos(\theta_r + k\Delta l) \\ B_2 = a_i \sin(\theta_i + k\Delta l) + a_r \sin(\theta_r + k\Delta l) \end{cases}$$
(11)

where $\theta_i = kx_1 + \varepsilon_i$, $\theta_r = kx_1 + \varepsilon_r$, *k* is the wave number, σ is the angular frequency and ε is the phase angle, the subscript "i" and "r" denotes incident and reflected waves, respectively. Δl is the spacing between two measuring stations.

The amplitudes a_i and a_r can thus be calculated by:

$$a_{i} = \frac{1}{2|\sin k\Delta l|} [(A_{2} - A_{1}\cos k\Delta l - B_{1}\sin k\Delta l)^{2} + (B_{2} + A_{1}\sin k\Delta l - B_{1}\cos k\Delta l)^{2}]^{1/2}$$

$$a_{r} = \frac{1}{2|\sin k\Delta l|} [(A_{2} - A_{1}\cos k\Delta l + B_{1}\sin k\Delta l)^{2} + (B_{2} - A_{1}\sin k\Delta l - B_{1}\cos k\Delta l)^{2}]^{1/2}$$
(12)
(13)

The energies of the incident and reflected waves, E_i and E_r could be obtained by:

$$E_{i} = \int_{f_{\min}}^{f_{\max}} S_{i}(f) df \tag{14}$$

$$E_{\rm r} = \int_{f_{\rm min}}^{f_{\rm max}} S_{\rm r}(f) df \tag{15}$$

 $S_i(f)$ and $S_r(f)$ denotes the frequency spectrum of incident and reflective waves, and *f* is the frequency. Consequently, the reflection coefficient, K_r , can be determined:

$$K_{\rm r} = \sqrt{\frac{E_{\rm r}}{E_{\rm i}}} \tag{16}$$

RESULTS AND DISCUSSION

Wave reflux and countercurrent

As shown in Fig. 3, when the waves pass through the series of submerged breakwaters, the sinusoidal dike-shape of the breakwater causes the former wave to propagate over the submerged breakwater. The wave is partially hindered and obstructed by the embankment surface. The wave potential reduced and the water surface is uplifted, hence, partially backflow of the wave are slip contrarily and affect the incoming incident rear waves.

Similarly, under the impact of waves-wave and wave-obstacle interactions, the rear incident-waves break before reaching the obstacle and above the obstacles, the occurrence of wave breaking can be found. Fig. 4 shows the phenomenon of surf wave in front and above the breakwater for an irregular wave case, which the attenuation of wave energy is greatly affected, the appropriate combinations of serious submerged breakwaters can accomplish the achievement of coastal protection, this phenomenon was also found in regular wave cases.



Fig. 3 Contrarily backflow wave $(w/h = 0.5 \cdot D/h = 0.75)$



Fig. 4 Surf wave (a) infront and (b) above breakwater ($w/h = 0.5 \cdot D/h = 0.75$, $T_{1/3} = 1.5$ sec)

Diversity of K_r variety relative to breakwater height, D/h

The variation of reflection coefficient K_r of a single row of sinusoidal submerged obstacle is first discussed, Fig. 5 shows the variation of Kr with w/h = 0.5 within dimensionless frequency $\sigma^2 h/g = 0.36 - 3.2$, and the relative breakwater heights D/h are 0.25, 0.5, 0.75 and 1.0, previously. $\sigma^2 h/g$ is a simplified form of the dispersion relations $\sigma^2 =$ gktanhkh which represents the interrelations of wave properties like wavelength, frequency, wavenumber for surface gravity wave on water of arbitrary, but constant depth, this term is used in the following discussion. When D/h = 0.25, the reflection coefficient K_r is ranges between 0.2 and 0.6, the reflection coefficient of a comparatively shortperiod wave is greater than that of long period wave, and K_r increases progressively with the increment of dimensionless frequency, a slight fluctuation appears on the variation of the reflection coefficient along $\sigma^2 h/g$ between 0.32 and 2.0. When D/h = 0.5, K_r ranged from 0.15 to 0.6, the maximum value of K_r for a long-period wave approached close to that of a short-period, and the concussion range of reflectivity for a long-period wave is comparatively apparent to that when D/h = 0.25. The fluctuation appeared stronger intense on the variation of the reflection coefficient along the foregoing range $\sigma^2 h/g = 0.32 - 2.0$. When D/h = 0.75, the maximum value of the reflection coefficient K_r of the

long-period wave is similar to that of the short-period wave, and ranged between 0.1 and 0.65. The fluctuation range of reflectivity for a longperiod wave is even more apparent than that of D/h = 0.25 and 0.5. However, this phenomenon ceases when D/h = 1.0, where the impact of the sinusoidal affection is no longer exists since most of the waves are reflected by the front of the barrier, and are partially overtopping.

The effects of wave height on the reflectivity are similar to the results when D/h = 0.25, the regularity on the variation of K_r values in shortperiod waves is greatly reduced. In summary, the relative reflectance values of the wave height variation H/h are generally 0.05 > 0.1 > 0.15 > 0.2, and the regularity of these values is not obvious in long-period and short-period waves. The above phenomenons are also similar in the case with multi-row breakwaters.



Fig.5 Variations of the reflection coefficient K_r for a single row of sinusoidal breakwater with $w/h = 0.5 \cdot D/h = 0.25 \cdot 0.5 \cdot 0.75$ and 1.0, under regular waves

Fig. 6 shows the variations of reflection coefficient K_r for a single row of sinusoidal breakwater of irregular wave test with w/h=0.5, relative to breakwater height D/h. The variation of the wave reflection indicates the reflection tendency rises slightly with the increasing in $\sigma^2 h/g$ when D/h = 0.25, and furthermore, the overall reflectance ratio of D/h = 0.75 and 1.0 is higher than that of D/h = 0.25 and 0.5. The results also revealed that the variations of K_r are apparently dissimilar compared to that of a monochromatic wave, the fluctuation phenomenon of the reflectance variation was not found, mainly because a random wave contains modulated waves, with various wave frequencies, wave heights

and wavelength, and consequently weakened the characteristics of a specific wave. It also exhibited from the figures that when the height of the submerged breakwater is the same as that of the water surface, the value of the reflection coefficient is much larger than in the other three cases, i.e. D/h = 0.25 - 0.75, regardless of barrier heights, the attenuation is limited under longer wavelength. The reflection coefficients are ranged between 0.55 and 0.95 when $\sigma^2 h/g = 0.36 - 1.5$, and 0.3 to 0.9 when $\sigma^2 h/g = 1.5 - 3.2$ for regular wave case, the average reflectance values are ranged between 0.7 and 0.9 for an irregular waves case.



Fig.6 Variations of the reflection coefficient K_r for a single row of breakwater with w/h = 0.5, D/h = 0.25, 0.5, 0.75 and 1.0, under irregular waves

Diversity of K_r variety relative to breakwater width, w/h

As shown in Fig. 7 when w/h = 0.5, the reflection coefficients K_r within $\sigma^2 h/g = 0.36-1.5$ (comparatively long wave) range between 0.5 and 0.95 under the condition of D/h = 1.0 and H/h = 0.05 - 0.2. These results show a slight reduction in variations, and the variances in reflectivity are inconspicuous in their entirety among various wave heights. For w/h= 1.0, the reflection coefficients K_r within $\sigma^2 h/g = 0.36-1.5$ range between 0.4 and 0.95, and the scattering range increases slightly. When the sinusoidal breakwater's width increases to w/h=1.5, the frequency of K_r fluctuation increases obviously. Since the dike high D/h = 1.0 is at the same level as the water surface, the response should theoretically be similar to the effect of a vertical quay, but as the wave height is above the water level, also with various wave dynamics based on different wave conditions. Part of the wave will overtops the dike, and affect the reflectivity, thus, K_r ranges between 0.3 and 0.9 within $\sigma^2 h/g = 1.5-3.2$ (comparatively short wave). Accordingly, this variation in K_r values shows that the scattering range of the reflectance values slightly reduced, and the variety of K_r substantially alleviated as gradually extended to a comparatively long-period waves.

The fluctuation of K_r values may have a close relation with the resonance of the incoming wave with the structure (say, Bragg resonance). It is necessary to analyze the correlation between the wavelengths of incident waves due to various dimensionless frequencies and the widths of the embankments, whether these correlations relate with these extreme values, which appear larger or smaller in particular locations. Unfortunately, these results have not yet presented a clear regular causality.

Fig.8 shows the variation of those in irregular wave tests. When w/h=0.5, the reflectance is slightly greater than that of w/h=1.0 and 1.5, this may due to the shape-steepness D/w of the sinusoidal breakwater, i.e. steeper. The reflectance of a shorter wave is greater than that of the a longer wave, but with the increasing of the breakwater height, the reflectance of a long-wave is contrarily better than that of a short-wave.



Fig.7 Variations of the reflection coefficient K_r for a single row of sinusoidal breakwater with D/h = 1.0, $w/h = 0.5 \times 1.0$ and 1.5, under regular waves



Fig.8 Variations of the reflection coefficient K_r for a single row of sinusoidal breakwater with $D/h = 1.0 \cdot w/h = 0.5 \cdot 1.0$ and 1.5, under irregular waves

Diversity of K_r variety with series breakwaters

Fig. 9 shows the variation of breakwater reflectance under the conditions of D/h = 0.75, and w/h = 0.5, 1-5 rows of series breakwaters, in relatively shallow waves with a longer wavelength range (e.g., $\sigma^2 h/g$ < 1.5). The effect of the change in wave height is not obvious, but the phenomenon of fluctuation on K_r value is apparent. Moreover, the scope of the incident wave is obviously affected by the row number of the breakwaters, for example, in the case of three rows, the fluctuating reflectance values K_r slowly converge toward 0.4. Conversely, the heights of incident waves influenced gradually the K_r values of waves in the ambit between shallow water waves with a shorter wavelength and intermediate waves (e.g., $\sigma^2 h/g > 1.5$), and gradually increased with the increasing of dimensionless frequency $\sigma^2 h/g$, namely the decreasing of wavelength-to-depth ratio L/h. Furthermore, the phenomenon of fluctuation on the K_r values is inconspicuously exhibited, and the impact on wave dissipation by the row number of sinusoidal obstacle is relatively small.

The reflectivity of five rows of breakwaters under irregular wave (Fig. 10) are compared with the previous results for regular wave displayed in

Fig. 9, the overall tendency of variations in reflectance between one and five rows are substantially parallel. However, the divergence of reflectivity due to different number of rows is inconspicuous, therefore the effect of row number of the submerged breakwater on the reflectivity for irregular wave is insignificance.

 $K_{\rm r}$ increased to larger quantitative values for longer waves under the condition of five Rows, the values is resembled in D/h = 0.25 and 0.50. As the breakwater height raised up to D/h = 0.75, although only a slight variation exists in five Rows, the quantity of such obstacle affect apparently in reflectance. Besides, the physical test for D/h = 1.0 in five Rows are not conducted.



Fig.9 Variations of the reflection coefficient K_r under various rows of sinusoidal breakwater with D/h = 0.75, w/h = 0.5 (regular waves)





CONCLUSIONS

This article discussed the impact of regular wave and irregular wave on different sinusoidal breakwater due to various heights, widths and quantity of rows. The reflectance characteristics with various combinations and varieties of breakwaters are investigated and compared. The phenomenon of waves passing through such sinusoidal breakwaters are discussed by the variation of reflectance. Some conclusions may be drawn from the present results as follows:

For a single-column breakwater, the tendency and fluctuating range of K_r values under long wave affections increases as the dike height increases, and conversely trends toward convergence and agglomeration when the dike width increases. However, this phenomenon was manifested quite reversely when the dike height is equal to the water depth (i.e., D/h=1.0).

The reflectance of a series of breakwaters is discuss in combination of two to five rows, however, the variation of reflection coefficient K_r is more complex. The tendency of fluctuating range of K_r values gradually converges to a gentle ambit as the row number of breakwaters and relative breakwater widths increases. For regular waves, this trend is similar to that when the widths of the embankment increases, yet the increasing trend are not quite regular. Under the conditions of the same width w/h and same arrangement of breakwaters, with various heights D/h, the variation of K_r , is similar to that of a single row of breakwaters. The results also revealed that the variations of K_r for irregular wave are apparently dissimilar compared to that of a monochromatic wave, the fluctuation phenomenon of the reflectance variation was not found.

Backflow wave slip conversely and interact the incident rear waves, waves are break before and above the breakwaters. This phenomenon was found in both the regular wave and irregular wave cases, and especially when the wavelengths (periods) are comparatively long.

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