THEORY AND APPLICATIONS OF BOUNDARY ELEMENT METHODS
Proceedings of the **Eighth China-Japan Symposium on BEM**, *May 11-15, 1998, Beijing, China*

DEVELOPMENT OF NUMERICAL IRREGULAR WAVE MAKING CHANNEL

Chung-Ren Chou and Kwan Ouyang***

* *Dept. of Harbor and River Eng., National Taiwan Ocean University, Keelung, Taiwan, China*

** *Dept. of Telecommunication Eng., China Junior College of Marine Technology, Keelung, Taiwan, China*

ABSTRACT A numerical scheme is developed in this study to simulate irregular waves. To generate waves, the power spectrum defined by significant wave height and significant wave period is employed as the condition of incident waves. The power spectrum of Brestschneider-Mitsuyasu type is chosen in this study. Based on the Lagrangian description and finite difference of time derivative, the generation and propagation of irregular waves are simulated numerically by means of boundary element method. The numerical model is verified by studying the case of irregular waves propagating the water tank with constant depth. The time histories of water elevation of measured points on free water surface are presented in this paper. Besides, the time histories of water profiles at different time steps are shown as well.

INTRODUCTION

Since the improved prediction technique of the extreme wave load can give a more reliable data for design of the offshore structures, the development of two-dimension numerical tank have been studied by many researchers.

Based on the linearized governing equations, Madsen (1970) simulated the periodical waves generated by piston-type wave-maker. Some numerical solutions of a variable-coefficient Korteweg-de Vries equation are derived by Johnson (1972) to describe the solitary wave moving onto a shelf. Faltinsen (1978) studied the sloshing in a water tank by means of the boundary integral method. Using the nonlinear initial boundary condition and velocity potential, Nakayama(1983) applied a boundary element technique to the analysis of nonlinear water wave problem. Based on the boundary integral equation, Brorsen and Larsen (1987) presented a new approach to the generation of wave for nonlinear regular waves. Using boundary element method, the generations, propagation, and deformations of solitary waves in the water tank are studied by Okamura and Yakuma (1987), Sugino and Tosaka (1990). With the Green's theorem, a time-domain second-order method for simulation of nonlinear wave propagation in a flume was developed by Isaacson (1994). Based on the Lagrangian description with time derivative, Chou and Shih (1996.a) analyzed the generation and deformation of solitary wave by means of boundary element method.

However, natural waves are known to be irregular. For the development of numerical simulation of nonlinear irregular waves, using Green-Naghdi theory, the nonlinear irregular wave simulation in a three-dimensional numerical water tank was reported by Xu and Pawlowski (1993). Boo and Kim (1994) presented the simulation of nonlinear irregular waves using a higher order boundary element method. Again, Boo and Kim (1997) employed the Stokes second-order nonlinear irregular waves as incident waves to analyze the problem of nonlinear diffraction acting on a floating structure.

In this study, wave spectrum defined by significant wave height $H_{1/3}$ and significant wave period $T_{1/3}$ is employed as the condition of incident irregular wave. Using the boundary element method with linear element, the propagation of irregular wave is simulated. From the numerical results, the time histories of free water elevations of measured points on free water surface are presented. The time histories of water profiles at different time steps are also shown as well.

NUMERICAL SIMULATION

The procedures of the numerical scheme in this paper are divided into three parts. The first is the setting of the initial condition for generating irregular wave; secondly, the irregular wave is simulated by means of the boundary element method; finally, the numerical results are compared with incident wave to assess this scheme.

1. The setting of the initial condition for generating irregular wave

Using the significant wave height $H_{1/3}$ and significant wave period $T_{1/3}$, the initial conditions for generating irregular wave can be given by employing three power spectrum defined below or arbitrary power spectrum. This three power spectrum can be expressed as

$$S_0(f) = Af^{-5}\exp\left(-Bf^{-4}\right) \tag{1}$$

While the power spectrum of Brestschneider-Mitsuyasu type is chosen,

$$A = 0.257\,H_{1/3}^{2}/T_{1/3}^{4}$$
$$B = 1.03T_{1/3}^{-4} \tag{2}$$

While the power spectrum of modified Brestschneider-Mitsuyasu type is chosen,

$$A = 0.205\,H_{1/3}^{2}/T_{1/3}^{4}$$
$$B = 0.75T_{1/3}^{-4}$$

While the power spectrum of Jonswap type is chosen,

$$A = \frac{0.0624}{0.23 + 0.0336\gamma - 0.185(1.9+\gamma)^{-1}}(1.094 - 0.01915\ln\gamma)\gamma\exp\left[-(f/f_p)^2/2\sigma^2\right]$$
$$B = 1.25f_p^{-4}$$
$$f_p = \frac{1}{T_p}$$
$$T_p = \frac{T_{1/3}}{1 - 0.132(\gamma+0.2)^{-0.559}}$$
$$\sigma = \begin{cases} 0.07 & f \le f_p \\ 0.09 & f \ge f_p \end{cases}$$
$$\gamma = 1 \sim 7 \tag{3}$$

where the T_p is peak period, f_p is peak frequency, γ is peak enhancement factor which the value is given 3.3 in this paper. Since the piston or flap type of wave-maker can be simulated in this numerical scheme, the power spectrum should be modified as

$$S(f) = \alpha(f)S_0(f) \tag{4}$$

where $\alpha(f)$ is the modified coefficient of wave-maker. For the piston type,

$$\alpha(f) = \frac{\sinh kh \cosh kh + kh}{\sinh^2 kh} \tag{5}$$

; and for the flap type,

$$\alpha(f) = \frac{\sinh kh \cosh kh + kh}{\sinh^2 kh} \tag{6}$$

where k is the wave number and h is depth of water tank. Ignoring the waves which period are too long or short, the wave period are limited as

$$T_{\min} < T < T_{\max} \tag{7}$$

$T_{\max} = 25$ seconds and $T_{\min} = 5$ seconds are given in this study.

According to the power spectrum, the water elevation $\zeta(t)$ can be obtained by

$$\zeta(t) = \sum_{n=1}^{\infty} \sqrt{2dfS(f_n)} \cos(\sigma_n t - \varepsilon_n) \tag{8}$$

where $\sigma_n = 2\pi f_n$, ε_n denotes the random number between 0 and 2π. The horizontal velocity of water particle at arbitrary depth, $U(t)$, can be expressed as

$$U(t) = \sum_{n=1}^{\infty} \sqrt{2dfS(f_n)}\sigma_n \frac{\cosh k(z+h)}{\cosh kh} \cos(\sigma_n t - \varepsilon_n) \tag{9}$$

2. Simulation of wave generation

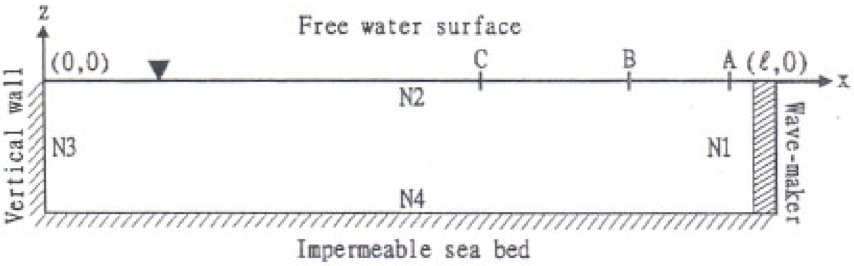

Fig.1 Definition of water tank

As shown in Fig.1, the original coordinate is located on the still water surface with x-axis positively right and z-axis positively upwards. The flow field is bounded by a pseudo wave-maker boundary Γ_1, a free water surface Γ_2, impermeable vertical wall and seabed, Γ_3, Γ_4 respectively. For inviscid and incompressible flow, the flow motion has velocity potential $\Phi(x,z;t)$ satisfying the Laplace equation as follows if it is irrotational.

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \tag{10}$$

Assuming that the atmospheric pressure on free water surface is equal to zero, the following relationships can be obtained from the kinematic and dynamic conditions.

$$u = \frac{Dx}{Dt} = \frac{\partial \Phi}{\partial x} \tag{11}$$

$$w = \frac{Dz}{Dt} = \frac{\partial \Phi}{\partial z} \tag{12}$$

$$\frac{D\Phi}{Dt} + g\eta - \frac{1}{2}\left[\left(\frac{\partial \Phi}{\partial x}\right)^2 + \left(\frac{\partial \Phi}{\partial z}\right)^2\right] = 0 \tag{13}$$

where D is the Lagrangian differentiation, g is the gravitational acceleration and η is the water surface elevation.

Since the boundaries of vertical and seabed are assumed to be impermeable, the fluid velocity normal to the boundaries are equal to zero, we thus have

$$\frac{\partial \Phi}{\partial n} = 0 \tag{14}$$

where n denotes the direction normal to boundary.

For continuity, the horizontal velocity of wave-making paddle, $U(t)$, and the fluid velocity have the following relationship:

$$\overline{\Phi} = \frac{\partial \Phi}{\partial n} = -U(t) \tag{15}$$

According to Green's Second Identity, the velocity potential at any point within the computing domain, $\Phi(x,z;t)$, can be obtained by the velocity potential on the boundary, $\Phi(\xi,\eta;t)$, and its normal derivative, $\partial\Phi(\xi,\eta;t)/\partial n$. The relationship is

$$\Phi(x,z;t) = \frac{1}{2\pi}\int_\Gamma \left[\frac{\partial \Phi(\xi,\eta;t)}{\partial n}\ln\frac{1}{r} - \Phi(\xi,\eta;t)\frac{\partial}{\partial n}\ln\frac{1}{r}\right]ds \tag{16}$$

where $r = \left[(\xi - x)^2 - (\eta - z)^2\right]^{1/2}$. When the inner point (x,z) is very close to the boundary point (ξ',η'), the velocity potential of that point, $\Phi(\xi',\eta';t)$, can be expressed as

$$\Phi(\xi',\eta';t) = \frac{1}{\pi}\int_\Gamma \left[\frac{\partial \Phi(\xi,\eta;t)}{\partial n}\ln\frac{1}{R} - \Phi(\xi,\eta;t)\frac{\partial}{\partial n}\ln\frac{1}{R}\right]ds \tag{17}$$

where $R = \left[(\xi - \xi')^2 + (\eta - \eta')^2\right]^{1/2}$.

In order to proceed the numerical calculation, referring Fig.1, the boundaries Γ_1 through Γ_4 are divided into N1 to N4 discrete linear elements respectively. With the introduction of local dimensionless coordinate, the integral equation can be expressed in matrix form:

$$[\Phi_i] = [O_{ij}][\overline{\Phi}_i] \qquad i,j = 1 \sim 4 \tag{18}$$

where $[\Phi]$ and $[\overline{\Phi}]$ denote the velocity potential on boundaries and its derivative, and $[O]$ is the related shape function.

Substituting initial boundary conditions into Eq.18, the velocity potential on pseudo wave-maker boundary, Φ_1^k, the normal derivative of velocity potential on free water surface, $\overline{\Phi}_2^k$, and the velocity potential on fixed impermeable boundaries, Φ_3^k, Φ_4^k, can be obtained. Differentiating the time derivative in Eq.11, Eq.12 and Eq.13 by using forward-difference, we can obtain the new positions of free water surface, x^{k+1}, z^{k+1}, and velocity potential on free water surface, Φ_2^{k+1}, at next time, $t=(k+1)\Delta t$.

$$\Phi_2^{k+1} = \Phi_2^k + \frac{1}{2}\left[\left(\frac{\partial \Phi_2}{\partial s}\right)^2 + \left(\frac{\partial \Phi_2}{\partial n}\right)^2\right]^k \Delta t - g z^{k+1} \Delta t \tag{19}$$

Repeating to use the boundary conditions and velocity potential on free water surface at next time to resolve Eq.18, the other unknown values, $\overline{\Phi}_1^{k+1}, \overline{\Phi}_3^{k+1}, \overline{\Phi}_4^{k+1}$ can be obtained. It can be expressed as a matrix form:

$$\begin{bmatrix} \Phi_1 \\ \overline{\Phi}_2 \\ \Phi_3 \\ \Phi_4 \end{bmatrix}^{k+1} = \begin{bmatrix} I & -O_{12} & 0 & 0 \\ 0 & -O_{22} & 0 & 0 \\ 0 & -O_{32} & I & 0 \\ 0 & -O_{42} & 0 & I \end{bmatrix}^{-1} \begin{bmatrix} O_{11} & 0 & O_{13} & O_{14} \\ O_{21} & -I & O_{23} & O_{24} \\ O_{31} & 0 & O_{33} & O_{34} \\ O_{41} & 0 & O_{43} & O_{44} \end{bmatrix} \begin{bmatrix} \overline{\Phi}_1 \\ \Phi_2 \\ \overline{\Phi}_3 \\ \overline{\Phi}_4 \end{bmatrix}^{k+1} \tag{20}$$

where "I" is the unit matrix. The procedures of numerical calculation are discussed in detail by Chou & Shih (1996.b).

3. Analyze the irregular waves

According to the water surface profiles obtained by using boundary element method, the mean elevation, significant wave height and significant wave period of irregular waves can be obtained by means of zero-up method. To verify the accuracy of this numerical model, we then use Fast Fourrior Transfer (FFT) to get the power spectrum and compare with that of incident wave.

NUMERICAL RESULTS

In this study , the depth and length of numerical water tank are $h = 100$ m, $l = 50h$. The power spectrum of Brestschneider-Mitsuyasu type is chosen and significant wave height $H_{1/3} = 2.5m$, significant wave period $T_{1/3} = 8$ seconds. Dividing the power spectrum into $N = 512$, the sampling time $dT = 0.5$, and the frequency $df = 1/NdT = 1/256$, this power spectrum is shown in Fig.2. The modified power spectrum for pseudo wave-maker of piston type is shown in Fig.3. The time histories of free water surface elevation are shown in Fig.4.

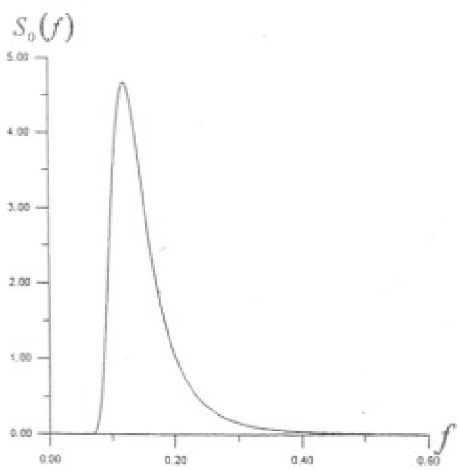

Fig.2 Power spectrum

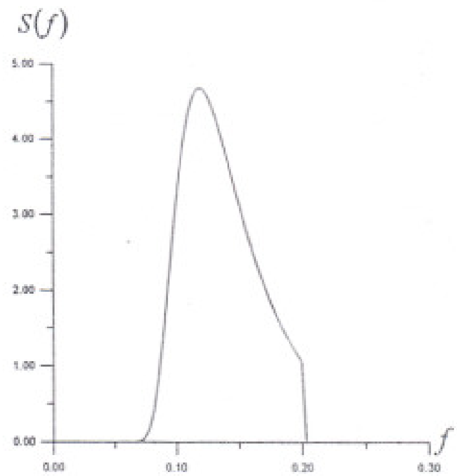

Fig.3 Corrected power spectrum

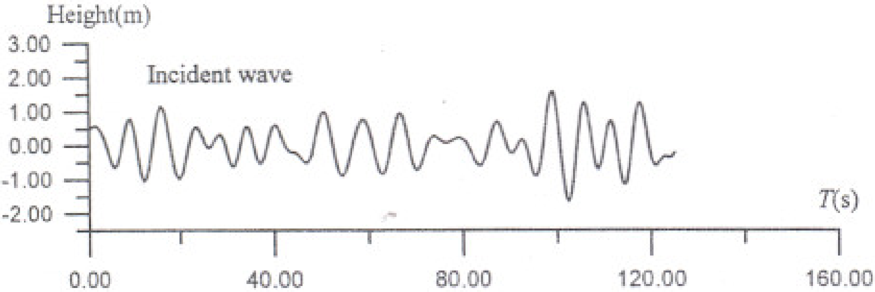

Fig.4 Time histories of free water surface elevation for incident wave

While simulate the generation of irregular waves, the element numbers on different boundaries are N1=6, N2=101, N3=6, N4=51, and the interval time of wave making $\Delta t = 0.05$ seconds.

Three measured points on free water surface are used to record the water surface elevation in this study. These points locate at the third, 10th, 20th node measured from pseudo wave-maker boundary and marked as A, B and C respectively. The time histories of free water surface elevation on different measured points are shown in Fig5.

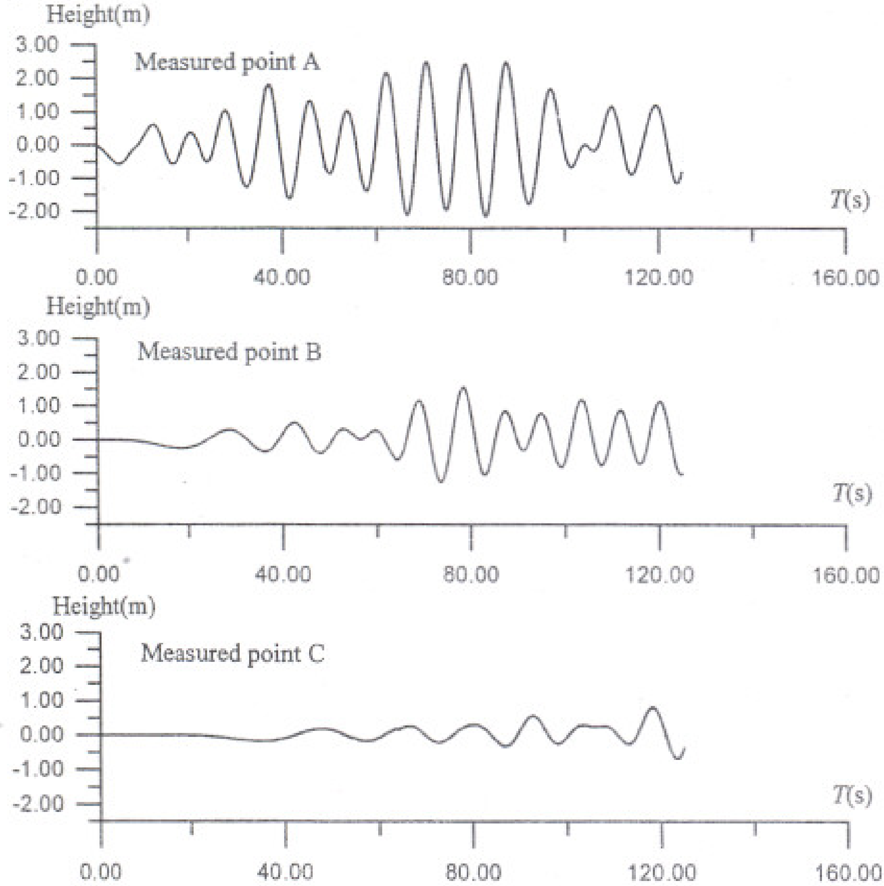

Fig.5 Time histories of water surface elevation on different points

Fig.6 indicates the water profiles at different time steps: $t = 800\Delta t, t = 1200\Delta t, t = 1600\Delta t, t = 2000\Delta t$, and $t = 2400\Delta t$.

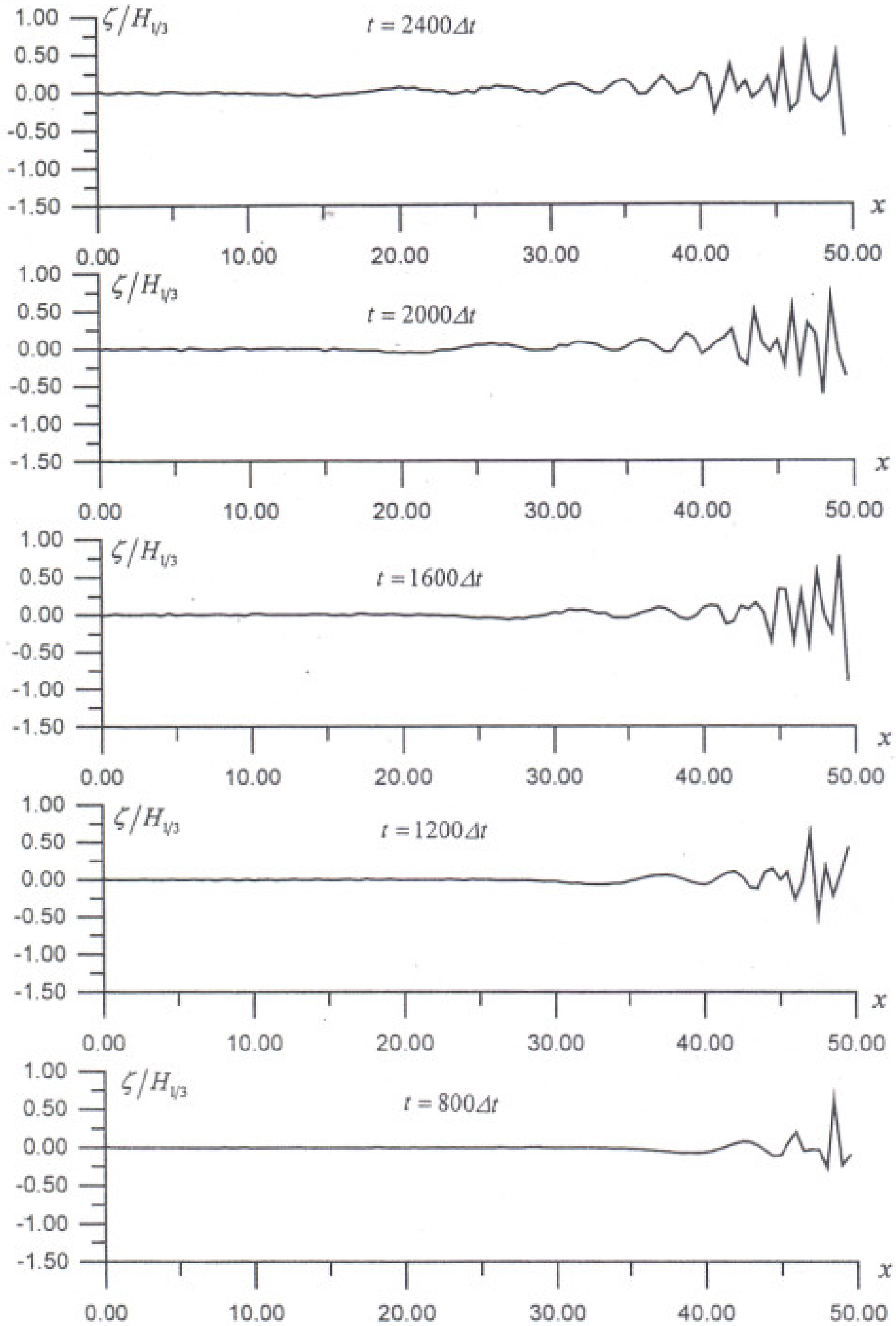

Fig.6 Water surface profiles at different time steps

CONCLUSION

This study is the case of simulation of two-dimensional numerical water tank for irregular wave by means of the boundary element method. To analyze the waves will be presented in future. There are two difficulty to develop the numerical water tank for simulating irregular wave. First, getting the stable numerical data is a point in issue. Additionally, it needs sufficient developing time and length to develop the irregular waves. That means the computing time will be longer and capacity of calculation will be larger. So, it is necessary to develop dissipation water tank in simulation of irregular.

REFERENCES

1. Boo, S. Y. and Kim, C. H., 1994, "Simulation of fully nonlinear irregular waves in a 3-D numerical wave tank", Pro. 4th International Offshore and Polar Engineering Conference, Vol. III, Osaka, Japan, pp.17-23.
2. Boo, S. Y. and Kim, C. H., 1997, "Nonlinear irregular waves and forces on truncated vertical cylinder", Pro. 7th International Offshore and Polar Engineering Conference, Vol. III, Honolulu, USA, pp.76-84.
3. Brorsen, M. and Larsen, J., 1987, "Source generation of nonlinear gravity waves with the boundary integral equation method", Coastal Engineering, Vol.11, No.4, pp.93-113.
4. Chou, C. R. and Shih, R. S., 1996.a. "Generation and deformation of solitary waves", China Ocean Engineering, China Ocean Press, Vo1.10, No.4, pp.419-432.
5. Chou, C. R., Shih, R. S. and Fang, H.M., 1996. "Defoemation of solitary wave in coastal zones", Pro. of the 7th Japan-China symposium on Boundary Element Method, Elsevier Press, pp.171~180.
6. Faltinsen, O., 1978, "A numerical nonlinear method of sloshing in tanks with two-dimensional flow", J. of ship research, Vol.22, No.3, pp.193-202.
7. Isaacson, M., Cheung, K. F., Mansard, E. and Miles, M. D., 1994, "Transient wave propagation in a laboratory flume", J. of Hydraulic Research, Vol.31, No.5, pp.665-680.
8. Johson, R. S., 1972, "Some numerical solutions of a variable-coefficient Korteweg-de Vries equation (with applications to solitary wave development on a shelf)", J. of Fluid Mech., No.54, pp.81~91
9. Madsen, O. S. and Mei, C. C., 1969, "The transformation of a solitary wave over an uneven bottom", J. of Fluid Mech., No. 39, pp. 781~791.
10. Nakayama, T., 1983, "Boundary element analysis of nonlinear water wave problem", International journal for numerical method in engineering, No.19, pp.953~970.
11. Sugino, R. and Tosaka, N., 1990, "Boundary element analysis of nonlinear water wave problem", Pacific Congress on Marine Science and Technology, pp.18~25.
12. Xu, Q., Pawlowski, J. S. and Baddour, R. E., 1993, "Simulation of nonlinear irregular waves by Green-Naghdi Theory", International Workshop on Water Wave and Floating Bodies, New-foundland, Canada.