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Ship motions near harbor caused by wave actions

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ABSTRACT: Around the harbour, waves affected by seabed topography as well as by breakwater could cause additional problems for ships sailing in this area. Motion of ships due to these waves was studied in this paper through a boundary element method. As verification of this method, wave-induced ship motions in the open seas were also calculated and compared with those of Ijima. The results are in satisfactory agreement.

1 INTRODUCTION

It is well known that ship motions are influenced by wave actions. Although studies of ship motions in the open sea have been carried out, but there are relatively few articles concerning the impact of wave forces upon ships around the harbor.

Near the harbor, waves are affected by the seabed topography. In addition, the interaction of incident and reflected waves, due to presence of breakwaters, can produce short crest waves in this area. Ship motions in this area are, therefore, plausibly different from those in the deep sea.

In this paper this problem is analyzed numerically. The ship is subjected to the action of waves, which are aroused through the presence of both seabed and breakwater. To simplified the analysis, the ship is taken to be rectangular in shape, and the harbor is enclosed by straight break-waters. Ship motions under the impact of waves having various periods and directions were studied. As a verification, this method is then applied to solve the open sea case. Comparison with the results obtained by other authors showed satisfactory agreement.

2 THEORETICAL ANALYSIS

Consider a region enclosed by the harbour, the breakwaters and the open sea, as shown schematically in Figure 1. Cartesian coordinates, (x,y,z), were chosen, with $z = 0$ the undisturbed free water surface, pointing positively upwards. Away from harbour, where wave scattering due to breakwater and harbour are negligibly small, an imaginary boundary, Γ_1, was drawn. The region under consideration were then be further divided into two sub-regions: an outer sea region, I, and a harbour region, II. The fluid is assumed to be inviscid, incompressible; wave motions are irrotational, and capillary effects are neglected.

Consider a small amplitude wave with a constant frequency, σ, ($=2\pi/T$, T is the wave period) and amplitude ζ_0 arriving from infinity. Fluid motions in these sub-regions will both have velocity potentials $\Phi(x,y,z;t)$, and is expressed as:

$$\Phi(x,y,z,t)=\frac{g\zeta_0}{\sigma}\phi(x,y,z)e^{-i\sigma t} \qquad (1)$$

where g is acceleration due to gravity, and $\phi(x,y,z)$ must satisfy the Laplace equation:

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0 \qquad (2)$$

2.1 Velocity potentials of the outer sea region

The outer sea region I, assumed to have a constant water depth, h, is enclosed by a boundary S_1, coast lines $\overline{AB}$, $\overline{GH}$ and a boundary at infinity. The velocity potential of this region can then be separated into two functions. The first one is depth dependent and is assumed to be known. The second, yet unknown, function depends on its location and is a result of superposition of the incident and reflected waves. The potential functions of this region can, therefore, be conveniently expressed as:

$$\phi_0(x,y,z) = \{f^0(x,y)+f^*(x,y)\}\frac{\cosh k(z+h)}{\cosh kh} \quad (3)$$

where k is the solution of $\sigma^2 h/g =$ kh tanh kh, f^0 and f^* are the potential functions of incident and reflected waves, respectively. The reflection is caused by the presence of the harbour, the ship and breakwaters.

When incoming waves bisect the x-axis with an angle ω, the resulting surface elevation, $\zeta_I(x,y;t)$, can be expressed as:

$$\zeta_I(x,y;t) = \zeta_0 \cos[k(x\cos\omega + y\sin\omega) + \sigma t], \quad (\pi \le \omega \le 0) \quad (4)$$

with the velocity potential:

$$f^0(x,y) = -i \cdot \exp[-ik(x\cos\omega + y\sin\omega)] \quad (5)$$

Substitution Eq. (3) into Eq. (2), one gets an equation for f^* which satisfies the Helmholtz equation with the form:

$$\frac{\partial^2 f^*}{\partial x^2} + \frac{\partial^2 f^*}{\partial y^2} + k^2 f^* = 0 \quad (6)$$

For the outer sea region, waves reflected from the coast lines $\overline{AB}$, $\overline{GH}$, can be taken as zero due to the assumption made. As for the reflected potential from the imaginary boundary at infinity, which satisfies the Sommerfeld radiation condition, can also be considered as vanishing small. The reflected potential function for any point, $f^*(x,y)$, within domain I can thus be found through the application of Green's integral technique, and is given as:

$$cf^*(x,y) = \int_{\Gamma_1} [f^*(\xi,\eta)\frac{\partial}{\partial\nu}(-\frac{i}{4}H_0^{(1)}(kR)) - (-\frac{i}{4}H_0^{(1)}(kR))\frac{\partial}{\partial\nu}f^*(\xi,\eta)]ds \quad (7)$$

where $f^*(\xi,\eta)$ is the potential function specified by the geometric boundary of domain I, $\partial f^*(\xi,\eta)/\partial\nu$ is its first normal derivative (directed positively outward). $H_0^{(1)}(KR)$ is the zeroth order Hankel function of the first kind, ν the unit normal vector. R is the distance between the point under consideration and the boundary. $R = \sqrt{(x-\xi)^2+(y-\eta)^2}$. Within the boundary, c equals to 1, but will have a value of 0.5 on the boundary, which are due the characteristics of the Hankel function.

In the following numerical analysis, the boundary Γ_1, where c = 1/2, is divided into N segments, each with constant element, and Eq. (7) is rewritten in the matrix form:

$$\{F^*\} = [K^*]\{\overline{F}^*\} \quad (8)$$

where $\{F^*\}$ is the potential function of the boundary and $\{\overline{F}^*\}$ its normal derivative. $[K^*]$ is a coefficient matrix, related to the shape of the geometric boundary (Chou, 1983).

2.2 Velocity potential of arbitrary water depth region

The harbour region II, with arbitrary water depth, is a closed three-dimensional domain. It is bounded by the imaginary boundary, Γ_1, the free surface, Γ_2, the immersed ship surface, Γ_3, an imaginary boundary of the basin, Γ_4, the breakwaters, Γ_5 and Γ_6, and an uneven sea bottom, Γ_7.

According to Green's second identity law, velocity potential of any point inside this region can be determined by velocity potential on the boundary and its first normal derivative, that is:

$$c\phi(x,y,z) = \int [\overline{\phi}(\xi,\eta,\zeta)(\frac{1}{4\pi}\frac{1}{R}) - \phi(\xi,\eta,\zeta)\frac{\partial}{\partial\nu}(\frac{1}{4\pi}\frac{1}{R})]dA \quad (9)$$

where $R = \sqrt{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2}$, and c = 1 for points inside the boundary and is equal to 1/2 on the boundary for the same reason mentioned before.

Dividing the surface of the boundaries, $\Gamma_1 \sim \Gamma_7$ into N_1 to N_7 discrete areas with constant element, the integral can then be transformed into a matrix form ready for calculation:

$$\{\phi\} = [K]\{\overline{\phi}\} \quad (10)$$

where both $\{\phi\}$ is the potential function of the boundary and $\{\overline{\phi}\}$ its normal derivative. [K] is a coefficient matrix, which is related to the shape of the geometric boundary (Chou, 1983).

2.3 The boundary conditions

The boundary conditions are summarized in the following:

1) The free surface condition, given by:

$$\overline{\phi} = \frac{\sigma^2}{g}\phi, \quad z = 0 \quad (11)$$

2) The condition at the sea floor:

$$\overline{\phi} = 0 \quad (12)$$

3) The requirements of continuity of mass and energy flux on the boundary Γ_1 is expressed as:

$$\overline{\phi_0}(\xi,\eta,\zeta) = \overline{\phi}(\xi,\eta,\zeta) \quad (13)$$

$$\phi_0(\xi,\eta,\zeta) = \phi(\xi,\eta,\zeta) \quad (14)$$

Together with Eq. (3), Eq. (13) yields an equation of the form:

$$\int_{-h}^{0} \overline{\phi}(\xi,\eta,\zeta)\cosh k(z+h)\,dz$$

$$=\int_{-h}^{0} (\overline{f}^{0}(\xi,\eta)+\overline{f}^{*}(\xi,\eta))\frac{\cosh^{2}k(z+h)}{\cosh kh}\,dz$$

Dividing the surface of the imaginary boundary, Γ_1 , into n segments vertically, and m segments horizontally, one then has m x n elements for the divided surface Γ_1. Normal potential of the reflected wave at any position on the boundary can then be calculated through:

$$\overline{f}^{*}(\xi_i,\eta_i)=$$

$$\frac{k}{N_0\sinh kh}\sum_{j=1}^{n}\overline{\phi}(\xi_i,\eta_i,\zeta_i)\cosh k(z_j+h)\Delta z_j$$

$$-\overline{f}^{0}(\xi_i,\eta_i)\ (i=1,2,\cdots\cdots,m) \tag{15}$$

where N_0 = 0.5(1+2kh/sinh 2kh).

Substitution of Eq. (3) into Eq. (14), one has:

$$\phi(\xi_i,\eta_i,\zeta_i)=$$

$$(f^{0}(\xi_i,\eta_i)+f^{*}(\xi_i,\eta_i))\frac{\cosh k(z_i+h)}{\cosh kh}$$

$$(i=1,2,\cdots\cdots,m\times n) \tag{16}$$

Combining Eqs. (8), (15) with Eq. (16), rearranging, the following matrix expression can be obtained:

$$\{\phi_1\}=[R]\{F^{0}-K^{*}\overline{F}^{0}\}$$
$$+c[R][K^{*}][Q]\{\overline{\phi}_1\} \tag{17}$$

where c = k/N_0sinh kh, subscript 1 denotes the boundary Γ_1, and [R], [Q] are coefficient matrices given by:

$$[R]=\begin{bmatrix}\frac{\cosh k(z_{11}+h)}{\cosh kh} & & \\ \vdots & & 0 \\ \frac{\cosh k(z_{1n}+h)}{\cosh kh} & \ddots & \\ & & \frac{\cosh k(z_{m1}+h)}{\cosh kh} \\ 0 & & \vdots \\ & & \frac{\cosh k(z_{mn}+h)}{\cosh kh}\end{bmatrix} \tag{18}$$

$$[Q]=\begin{bmatrix}\cosh k(z_{11}+h)\Delta z_1\cdots\cosh k(z_{1n}+h)\Delta z_n & \\ \ddots & \\ & \cosh k(z_{m1}+h)\Delta z_1\cdots\cosh k(z_{mn}+h)\Delta z_n\end{bmatrix} \tag{19}$$

4) By assuming that the breakwaters are impermeable, one has the boundary condition:

$$\overline{\phi}(\xi,\eta,\zeta)=0 \tag{20}$$

5) The main propose of this paper is to analyze wave induced ship motions near the harbour entrance. To simplify the problem, conditions within the harbour basin will not be considered. An imaginary boundary within the harbor basin, away from the entrance, is thus chosen. To save computing expanses, it is further assumed that, waves impinging Γ_4 will not be reflected back again. Under this circumstances, the boundary equation is similar to the Sommerfeld radiation condition, and is given by:

$$\overline{\phi}(\xi,\eta,\zeta)=ik\phi(\xi,\eta,\zeta) \tag{21}$$

6) Kinematic boundary condition on immersed ship surface

When subjected to wave actions, a ship displaces away from its center of gravity, $(\overline{x}_0,\overline{y}_0,\overline{z}_0)$, to a new position (x_0,y_0,z_0). The small quantities, as it rotates with respect to the (x,y,z) axis, are denoted as δ_1, δ_2, and δ_3. The position $\zeta(x,y;t)$ of the immersed ship surface can be calculated using:

$$\zeta=$$
$$-[(x-\overline{x}_0)+\delta_3(y-\overline{y}_0)-\delta_2(z-\overline{z}_0)]\zeta_x$$
$$-[(y-\overline{y}_0)+\delta_1(z-\overline{z}_0)-\delta_3(x-\overline{x}_0)]\zeta_y$$
$$+[(z-\overline{z}_0)+\delta_2(x-\overline{x}_0)-\delta_1(y-\overline{y}_0)] \tag{22}$$

since

$$d\zeta/dt=u\zeta_x+v\zeta_y+w\zeta_z+\zeta_t=0 \tag{23}$$

where u, v, and w are velocity vectors in the usual sense. Which can be rewritten in the form of a velocity potential as:

$$\Phi_x\zeta_x+\Phi_y\zeta_y-\Phi_z+\zeta_t=0 \tag{24}$$

where ζ_x, ζ_y and -1 can be treated as normal vectors of the submerged portion of the ship at equilibrium about the x, y, and z directions, respectively. The surface boundary condition can then be expressed as:

$$\frac{\partial\Phi}{\partial\nu}=\frac{\partial x_0}{\partial t}\frac{\partial x}{\partial\nu}+\frac{\partial y_0}{\partial t}\frac{\partial y}{\partial\nu}+\frac{\partial z_0}{\partial t}\frac{\partial z}{\partial\nu}$$
$$+\frac{\partial\delta_1}{\partial t}\left[(y-\bar{y}_0)\frac{\partial z}{\partial\nu}-(z-\bar{z}_0)\frac{\partial y}{\partial\nu}\right]$$
$$+\frac{\partial\delta_2}{\partial t}\left[(z-\bar{z}_0)\frac{\partial x}{\partial\nu}-(x-\bar{x}_0)\frac{\partial z}{\partial\nu}\right]$$
$$+\frac{\partial\delta_3}{\partial t}\left[(x-\bar{x}_0)\frac{\partial y}{\partial\nu}-(y-\bar{y}_0)\frac{\partial x}{\partial\nu}\right] \quad (25)$$

When subjected to the action of waves, the ship will oscillate in accordance with the frequency of the wave. Expressing the displacements of the ship body as: ξ^*, η^* and ζ^* and the rotations as ω_1^*, ω_2^*, and ω_3^*, one then has the following relationship between (x_0, y_0, z_0) and $(\bar{x}_0, \bar{y}_0, \bar{z}_0)$:

$$\left.\begin{aligned} x_0 &= \bar{x}_0+\xi^* e^{-i\sigma t}\\ y_0 &= \bar{y}_0+\eta^* e^{-i\sigma t}\\ z_0 &= \bar{z}_0+\zeta^* e^{-i\sigma t}\end{aligned}\right\} \quad (26)$$

$$\left.\begin{aligned} \delta_1 &= \omega_1^* e^{-i\sigma t}\\ \delta_2 &= \omega_2^* e^{-i\sigma t}\\ \delta_3 &= \omega_3^* e^{-i\sigma t}\end{aligned}\right\} \quad (27)$$

When a ship is displaced from equilibrium, there will gravity, dynamic pressure due to fluid motion, and the restoring force, R. The components in the x-, y-, z-direction will be denoted as R_x, R_y and R_z (where R_x = R_y and R_z (where $R_x = R_y = 0$ is assumed). In a similar way, the moments due to restoring force R can be expressed as M_x, M_y, and M_z (where $M_x = 0$). Then, to the first approximation, we have the following equations of motion:

$$\left.\begin{aligned} m\frac{d^2x_0}{dt^2} &= \iint_{\Gamma_3} p\frac{\partial x}{\partial\nu}d\Gamma\\ m\frac{d^2y_0}{dt^2} &= \iint_{\Gamma_3} p\frac{\partial y}{\partial\nu}d\Gamma\\ m\frac{d^2z_0}{dt^2} &= \iint_{\Gamma_3} p\frac{\partial z}{\partial\nu}d\Gamma+R_z\end{aligned}\right\} \quad (28)$$

$$\left.\begin{aligned} I_x\frac{d^2\delta_1}{dt^2} &= \iint_{\Gamma_3} p\left[\frac{\partial z}{\partial\nu}(y-\bar{y}_0)-\frac{\partial y}{\partial\nu}(z-\bar{z}_0)\right]d\Gamma+M_x\\ I_y\frac{d^2\delta_2}{dt^2} &= \iint_{\Gamma_3} p\left[\frac{\partial x}{\partial\nu}(z-\bar{z}_0)-\frac{\partial z}{\partial\nu}(x-\bar{x}_0)\right]d\Gamma+M_y\\ I_z\frac{d^2\delta_3}{dt^2} &= \iint_{\Gamma_3} p\left[\frac{\partial y}{\partial\nu}(x-\bar{x}_0)-\frac{\partial x}{\partial\nu}(y-\bar{y}_0)\right]d\Gamma\end{aligned}\right\} \quad (29)$$

$$R_z=-\iint_{\Gamma_3}\rho g z_0\frac{\partial z}{\partial\nu}d\Gamma \quad (30)$$

$$\left.\begin{aligned} M_x &= -\iint_{\Gamma_3}\rho g\delta_1(y-\bar{y}_0)\left[\frac{\partial z}{\partial\nu}(y-\bar{y}_0)-\frac{\partial y}{\partial\nu}(z-\bar{z}_0)\right]d\Gamma\\ M_y &= -\iint_{\Gamma_3}\rho g\delta_2(x-\bar{x}_0)\left[\frac{\partial x}{\partial\nu}(z-\bar{z}_0)-\frac{\partial z}{\partial\nu}(x-\bar{x}_0)\right]d\Gamma\end{aligned}\right\} \quad (31)$$

where m is the mass of the ship, and I_x, I_y, and I_z are the components of the inertial moment through the center of gravity. The restoring forces R_z and moments M_x, M_y are given by:

Together with

$$\left.\begin{aligned} \frac{\partial x_0}{\partial t} &= -i\sigma\xi^* e^{-i\sigma t}, & \frac{\partial\delta_1}{\partial t} &= -i\sigma\omega_1^* e^{-i\sigma t}\\ \frac{\partial y_0}{\partial t} &= -i\sigma\eta^* e^{-i\sigma t}, & \frac{\partial\delta_2}{\partial t} &= -i\sigma\omega_2^* e^{-i\sigma t}\\ \frac{\partial z_0}{\partial t} &= -i\sigma\zeta^* e^{-i\sigma t}, & \frac{\partial\delta_3}{\partial t} &= -i\sigma\omega_3^* e^{-i\sigma t}\end{aligned}\right\} \quad (32)$$

and using Eqs. (26) and (27), as well as the relation $p/\rho g\zeta_0 = i\phi\exp(-i\sigma t)$, Eqs. (28) and (29) can be rearranged yielding:

$$\left.\begin{aligned} \frac{\xi^*}{\zeta_0} &= -a_1\iint_{\Gamma_3} i\phi\frac{\partial x}{\partial\nu}d\Gamma\\ \frac{\eta^*}{\zeta_0} &= -a_2\iint_{\Gamma_3} i\phi\frac{\partial y}{\partial\nu}d\Gamma\\ \frac{\zeta^*}{\zeta_0} &= -a_3\iint_{\Gamma_3} i\phi\frac{\partial z}{\partial\nu}d\Gamma\end{aligned}\right\} \quad (33)$$

$$\left.\begin{aligned} \frac{\omega_1^*}{\zeta_0} &= -\alpha_1\iint_{\Gamma_3} i\phi\left[\frac{\partial z}{\partial\nu}(y-\bar{y}_0)-\frac{\partial y}{\partial\nu}(z-\bar{z}_0)\right]d\Gamma\\ \frac{\omega_2^*}{\zeta_0} &= -\alpha_2\iint_{\Gamma_3} i\phi\left[\frac{\partial x}{\partial\nu}(z-\bar{z}_0)-\frac{\partial z}{\partial\nu}(x-\bar{x}_0)\right]d\Gamma\\ \frac{\omega_3^*}{\zeta_0} &= -\alpha_3\iint_{\Gamma_3} i\phi\left[\frac{\partial y}{\partial\nu}(x-\bar{x}_0)-\frac{\partial x}{\partial\nu}(y-\bar{y}_0)\right]d\Gamma\end{aligned}\right\} \quad (34)$$

with

$$
\left.
\begin{aligned}
a_1 = a_2 &= \frac{\rho}{m\sigma^2 / g} \\
a_3 &= \frac{\rho}{\dfrac{m\sigma^2}{g} - \displaystyle\iint_{\Gamma_3} \rho \frac{\partial z}{\partial \nu} d\Gamma}
\end{aligned}
\right\} \quad (35)
$$

$$
\alpha_1 = \frac{\rho}{\dfrac{I_1\sigma^2}{g} - \displaystyle\iint_{\Gamma_3} \rho g (y-\bar{y}_0)\left(\frac{\partial z}{\partial \nu}(y-\bar{y}_0) - \frac{\partial y}{\partial \nu}(z-\bar{z}_0)\right) d\Gamma}
$$

$$
\alpha_2 = \frac{\rho}{\dfrac{I_2\sigma^2}{g} - \displaystyle\iint_{\Gamma_3} \rho g (z-\bar{z}_0)\left(\frac{\partial x}{\partial \nu}(z-\bar{z}_0) - \frac{\partial z}{\partial \nu}(x-\bar{x}_0)\right) d\Gamma}
$$

$$
\alpha_3 = \frac{\rho}{I_3\sigma^2 / g} \quad (36)
$$

Substituting Eq. (1) and Eqs. (32) ~ (36) into Eq. (25), then the velocity normal to the surface of the ship can be obtained as:

$$
\begin{aligned}
\frac{\partial \phi}{\partial \nu} = & \\
& -\frac{\sigma^2}{g}\Big\{ a_1 \frac{\partial x}{\partial \nu} \iint_{\Gamma_3} \phi \frac{\partial x}{\partial \nu} d\Gamma + a_2 \frac{\partial y}{\partial \nu} \iint_{\Gamma_3} \phi \frac{\partial y}{\partial \nu} d\Gamma \\
& + a_3 \frac{\partial z}{\partial \nu} \iint_{\Gamma_3} \phi \frac{\partial z}{\partial \nu} d\Gamma + \alpha_1 \left(\frac{\partial z}{\partial \nu}(y-\bar{y}_0) - \frac{\partial y}{\partial \nu}(z-\bar{z}_0)\right) \\
& \cdot \iint_{\Gamma_3} \phi \left(\frac{\partial z}{\partial \nu}(y-\bar{y}_0) - \frac{\partial y}{\partial \nu}(z-\bar{z}_0)\right) d\Gamma \\
& + \alpha_2 \left(\frac{\partial x}{\partial \nu}(z-\bar{z}_0) - \frac{\partial z}{\partial \nu}(x-\bar{x}_0)\right) \\
& \cdot \iint_{\Gamma_3} \phi \left(\frac{\partial x}{\partial \nu}(z-\bar{z}_0) - \frac{\partial z}{\partial \nu}(x-\bar{x}_0)\right) d\Gamma \\
& + \alpha_3 \left(\frac{\partial y}{\partial \nu}(x-\bar{x}_0) - \frac{\partial x}{\partial \nu}(y-\bar{y}_0)\right) \\
& \cdot \iint_{\Gamma_3} \phi \left(\frac{\partial y}{\partial \nu}(x-\bar{x}_0) - \frac{\partial x}{\partial \nu}(y-\bar{y}_0)\right) d\Gamma \Big\}
\end{aligned} \quad (37)
$$

The immersed ship surface is divided into N_3 elements for the calculation, and Eq. (37) is expressed in matrix form as:

$$
[\bar{\phi}] = \{k_i\}[\phi] \quad (38)
$$

where

$$
\{k_i\} = a_{ij} + b_{ij} + c_{ij} + d_{ij} + e_{ij} \quad (39)
$$

$$
(i, j = 1, 2, \cdots, N_3)
$$

and

$$
\left.
\begin{aligned}
a_{ij} = & \\
& -a_1 \frac{\sigma^2}{g}\left[\left(\frac{\partial x}{\partial \nu}\right)_j \left(\frac{\partial x}{\partial \nu}\right)_i + \left(\frac{\partial y}{\partial \nu}\right)_j \left(\frac{\partial y}{\partial \nu}\right)_i \right] d\Gamma_j \\
b_{ij} = & -a_3 \frac{\sigma^2}{g}\left[\left(\frac{\partial z}{\partial \nu}\right)_j \left(\frac{\partial z}{\partial \nu}\right)_i \right] d\Gamma_j \\
c_{ij} = & -\alpha_1 \frac{\sigma^2}{g}\left[\frac{\partial z}{\partial \nu}(y-\bar{y}_0) - \frac{\partial y}{\partial \nu}(z-\bar{z}_0)\right]_j \\
& \cdot \left[\frac{\partial z}{\partial \nu}(y-\bar{y}_0) - \frac{\partial y}{\partial \nu}(z-\bar{z}_0)\right]_i d\Gamma_j \\
d_{ij} = & -\alpha_2 \frac{\sigma^2}{g}\left[\frac{\partial x}{\partial \nu}(z-\bar{z}_0) - \frac{\partial z}{\partial \nu}(x-\bar{x}_0)\right]_j \\
& \cdot \left[\frac{\partial x}{\partial \nu}(z-\bar{z}_0) - \frac{\partial z}{\partial \nu}(x-\bar{x}_0)\right]_i d\Gamma_j \\
e_{ij} = & -\alpha_3 \frac{\sigma^2}{g}\left[\frac{\partial z}{\partial \nu}(x-\bar{x}_0) - \frac{\partial y}{\partial \nu}(y-\bar{y}_0)\right]_j \\
& \cdot \left[\frac{\partial z}{\partial \nu}(x-\bar{x}_0) - \frac{\partial y}{\partial \nu}(y-\bar{y}_0)\right]_i d\Gamma_j
\end{aligned}
\right\} \quad (40)
$$

2.4 The system of equations

Together with the boundary conditions, Eq. (10) yields a system of linear equations with respect to ϕ and $\bar{\phi}$:

$$
\begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \end{bmatrix}
=
\begin{bmatrix}
k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} & k_{17} \\
k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} & k_{27} \\
k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} & k_{37} \\
k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} & k_{47} \\
k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} & k_{57} \\
k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} & k_{67} \\
k_{71} & k_{72} & k_{73} & k_{74} & k_{75} & k_{76} & k_{77}
\end{bmatrix}
\begin{bmatrix} \bar{\phi}_1 \\ \bar{\phi}_2 \\ \bar{\phi}_3 \\ \bar{\phi}_4 \\ \bar{\phi}_5 \\ \bar{\phi}_6 \\ \bar{\phi}_7 \end{bmatrix}
\quad (41)
$$

Eq. (41) expresses the relations of the potential functions to their normal derivations on the boundaries. Using the assumptions of impermeable sea bottom and breakwaters, i.e., Equations. (12) and (20), one has an equation for the boundaries 1 ~ 4, expressed as:

$$\begin{bmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\end{bmatrix}=\begin{bmatrix}k_{11} & k_{12} & k_{13} & k_{14}\\ k_{21} & k_{22} & k_{23} & k_{24}\\ k_{31} & k_{32} & k_{33} & k_{34}\\ k_{41} & k_{42} & k_{43} & k_{44}\end{bmatrix}\begin{bmatrix}\bar{\phi}_1\\ \bar{\phi}_2\\ \bar{\phi}_3\\ \bar{\phi}_4\end{bmatrix} \qquad (42)$$

By use of Eqs. (11), (17), (21), and (38), oen can rewrite Eq. (42) in the following form:

$$\begin{bmatrix}(k_{11}-cRk^*Q) & \frac{\sigma^2}{g}k_{12} & k_{13}k_* & ikk_{14}\\ k_{21} & \frac{\sigma^2}{g}k_{22}-I & k_{23}k_* & ikk_{24}\\ k_{31} & \frac{\sigma^2}{g}k_{32} & k_{33}k_*-I & ikk_{34}\\ k_{41} & \frac{\sigma^2}{g}k_{42} & k_{43}k_* & ikk_{44}-I\end{bmatrix}\begin{bmatrix}\bar{\phi}_1\\ \phi_2\\ \phi_3\\ \phi_4\end{bmatrix}$$

$$=\begin{bmatrix}R(F^0-k^*\bar{F}^0)\\ 0\\ 0\\ 0\end{bmatrix} \qquad (43)$$

Equation (43) can be solved for the potential function on the boundaries 2 ~ 4, and the normal derivative of the velocity potential $\bar{\phi}_1$ on boundary Γ_1. The amplitude of the ship motion at the center of gravity can then be calculated using Eqs. (32) and (33).

2.5 Analysis for the case of open sea

As shown in Fig. 2, one can divide the open sea region into two sub-regions: one outer sea region with a constant water depth, I, and the another one having arbitrary water depth where the ship is present, II. For the outer sea region, I, the analytical method is similar to that given in section 2.1. The region with arbitrary water depth, region II, was enclosed by four boundaries: the imaginary boundary, Γ_1, the still water level, Γ_2, the immersed ship surface, Γ_3, and the impermeable sea bottom, Γ_4.

Following the procedures described in section 2.4, a set of linear equations can be derived:

$$\begin{bmatrix}(k_{11}-cRk^*Q) & \frac{\sigma^2}{g}k_{12} & k_{13}k_*\\ k_{21} & \frac{\sigma^2}{g}k_{22}-I & k_{23}k_*\\ k_{31} & \frac{\sigma^2}{g}k_{32} & k_{33}k_*-I\end{bmatrix}\begin{bmatrix}\bar{\phi}_1\\ \phi_2\\ \phi_3\end{bmatrix}$$

$$=\begin{bmatrix}R(F^0-k^*\bar{F}^0)\\ 0\\ 0\end{bmatrix} \qquad (44)$$

where $\bar{\phi}_1$ is the normal velocity potential on the imaginary boundary Γ_1, ϕ_2 the potential function of the free surface, and ϕ_3 the immersed ship surface.

The amplitudes of the ship motions can be obtained by substituting the solution of Eq. (44) into Eqs. (33) and (34).

3 THE NUMERICAL RESULTS AND DISCUSSION

In all the cases presented here, the ship was assumed to have the shape of a box for simplicity. The length is taken to be 2b/h = 4, the breadth is 2a/h = 0.8 and the draught is qh = 0.5. Figs. 3 and 4 show the responses of the ship to regular waves on open sea. The regular waves have nondimensional periods σ^2h/g = 0.25 and 0.5 and incoming angles ω = 0° ~ 90° with the x-axis. The results of Ijima for the same conditions were shown as dotted lines in Fig. 4 for comparison. As can be seen, except for sway, ζ^*, and roll, ω_2^*, which tend to be smaller, these two methods yield results which are in satisfactory agreement.

For the case of ships sailing around harbour, the same scales for the ship were used. The problem is further simplified by assuming the harbour is enclosed by infinity straight breakwaters. The entrance width is 4h (Fig. 5). Motions of the ship are calculated using nondimensional wave periods σ^2h/g = 0.25 ~ 0.75 and the directions are ω = 0° ~ 90°.

The imaginary boundary of the outer sea, shown in Fig. 5, is set at x = ± 5h and y= 8h. Under these circumstances, the effect of ship-induced wave scattering on the imaginary boundary, Γ_1, should thus be negligible. This boundary is then divided into: n = 2 vertically, and m = 52 horizontally, yielding a total of 104 elements. The free surface, Γ_2, is divided into square elements, having a length of 0.5h for each side. The sea floor is roughly divided into 190 elements. The imaginary boundary inside the harbour basin, Γ_4, located at x = ± 4h and y = -4h, is divided into 20 elements, each breakwater has 11 elements. As for the immersed ship surface, shown in Fig. 6, it was divided into 68 elements. The mass, m, and the inertial moments about the x-, y-, z-axes, Ix, Iy, Iz, were calculated using the following equations:

$$m = 4\rho abqh$$

$$I_x = \frac{4}{3}\rho\nu_1^2 ab^3qh\left(1+\left(\frac{qh}{2b}\right)^2\right)$$

$$I_y = \frac{4}{3}\rho\nu_2^2 a^3bqh\left(1+\left(\frac{qh}{2a}\right)^2\right)$$

$$I_z = \frac{4}{3}\rho\nu_3^2 abqh\left(a^2+b^2\right)$$

where ν_1^2, ν_2^2, and ν_3^2 are coefficients of the distribution of density of the ship, they were all assumed to be 1.25 in this paper.

When a ship in its longitudinal direction along the y-axis affected by the prescribed wave conditions, calculated results are presented in Fig. 7 through 15. With the ship's center of gravity located at (0,0), the results are shown in Figs. 7 ~ 9. Figs. 10 ~ 12 are for the case (0.2h), and Figs. 13 ~ 15 for (0.4h).

When waves propagate obliquely into a region where breakwaters are present, short crested waves will be produced. The characteristics of these depend heavily on the incident angle and period. Ship motions in this area should, therefore, differ from those in the open sea. The location of the ship is also an important factor for the behavior of the ship.

When center of gravity of the ship is located at (0,0), with a dimensionless period σ^2h/g = 0.25, sway ξ^* has its maximum when the incident angle ω is in the vicinity of 45°, tending to become smaller for decreasing wavelength. For the same wave period, the heave, ζ^*, on the other side, has its minimum at ω = 0°, increasing momotonously for larger incident angles and smaller wave lengths. However, after the appearance of the maximum, it has a tendency of decreasing for increasing incident angles. The minimum of the pitch ω_1^* occurs at ω = 0°, (zero for open sea), tends to increase for growing incident angles, and becomes larger for shorter waves. The roll, ω_2^*, is zero at ω = 90°, has a maximum value when the incident angle is around 45°. The surge, η^*, attains its peak at ω = 90°, decreasing for shorter waves. And finally the yaw, ω_3^*, reaches its peak value at ω = 45°, is hardly affected by the changes of wavelength. It behaves differently as for the case of open sea, where it should be zero for a zero angle of incidence.

When the center of gravity is located at (0,2h), the maxima of both sway, ξ^*, and heave, ζ^* increase clearly. The appearance of these maxima tends to occur at smaller angles. As for the other modes of the motion, there are no appreciable difference between this case and the previous one.

For the case (0,4h), the maxima of sway, ξ^*, and heave, ζ^*, are larger than for the case of (0,2h). The peaked values tend to occur at even smaller incident angles and for shorter waves. The pitch, ω_1^*, the yaw, ω_3^*, and the surge, η^*, all have their minimum values for incident angles ω in the range of 0° ~ 15°. For σ^2h/g = 0.75, they are zero.

It can thus be concluded, that when the incident angle is very small, i.e., when wave rays are almost parallel to the breakwater, and when the ship is far away from the breakwater, ship motions have a clear resemblance to those in the open sea. When the incident angle of waves is large enough to produce short crested waves, the position of its loops and nodes will depend on the incident angles, as well as on wave periods. In this paper, the assumption of infinite straight breakwaters is used, with the results of ship motions that change periodically in accordance with its position. Since areas which can produce short crested waves are, in fact, finite, the critical position(s) of the most severe wave-induced ship motions for specified wave conditions can be acquired through comparison.

4 CONCLUSION

A boundary element method is used to analyze wave-induced ship motions, when the ship is near a harbour entrance, or structures. Since the authors are unaware of relevant reference(s) for confirmation, an alternative was chosen. Ship motions for the case of open sea under the action of regular waves were analyzed and compared results from other authors. The agreements are satisfactory. Experiemnts were carried out for the case of ships sailing near harbour entrance. However, due to limited size of the plane tank, 24 x 30 m, stable short crested waves, and/or standing wave can not be produced successfully. Further confirmation must await for experiments to be carried out in our new plane tank in the near future.

5 REFERENCES

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Chou, C.R. & J. G. Lin 1989 "Numerical analysis of harbor oscillation with wave absorber" 11th Conf. Ocean Engng. Keelung, Rep. China October, pp. 111-129 (in Chinese)

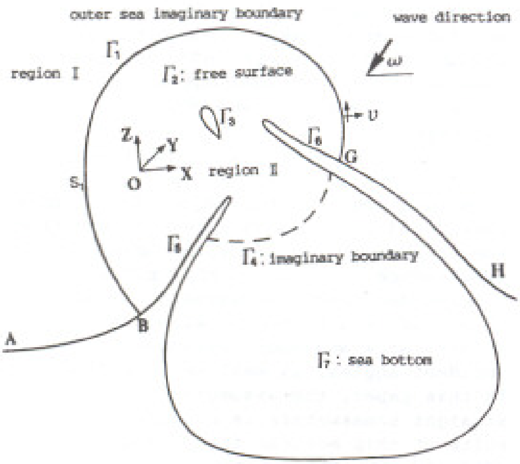

Fig. 1 Definition sketch

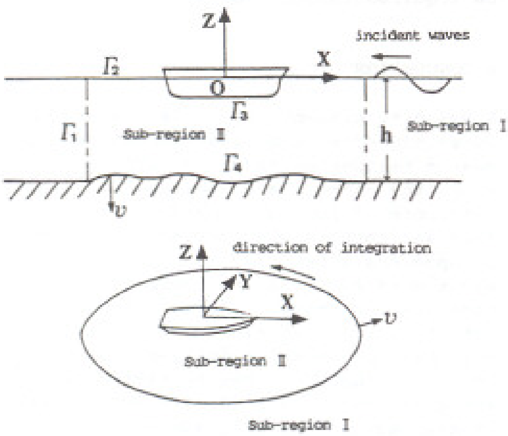

Fig. 2 Definition sketch for open sea

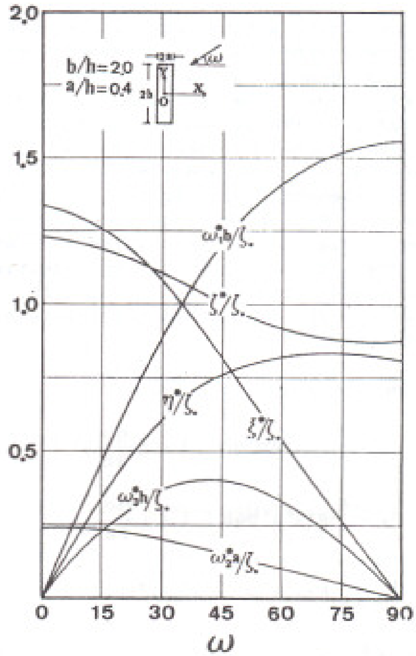

Fig. 3 Motions of a box-shaped ship in open sea (σ^2h/g=0.25, kh=0.522)

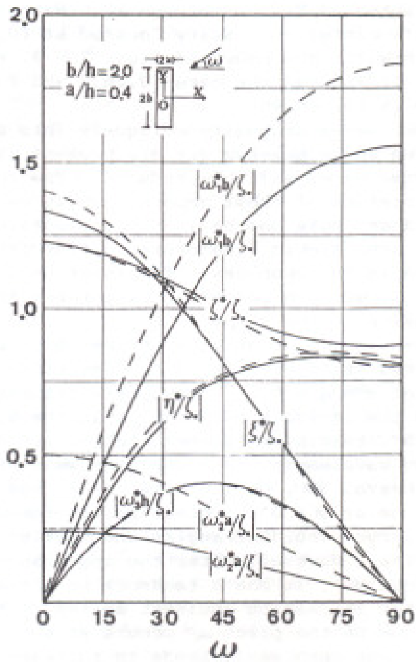

Fig. 4 Motions of a box-shaped ship in open sea (σ^2h/g=0.5, kh=0.772)

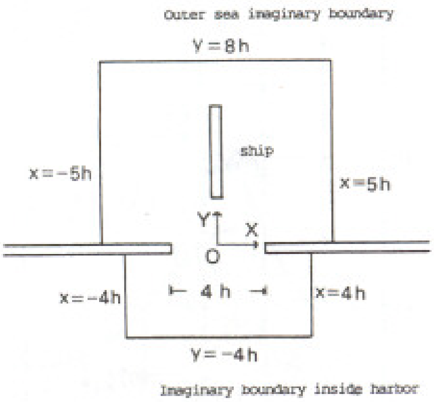

Fig. 5 Setups of the imaginary boundary

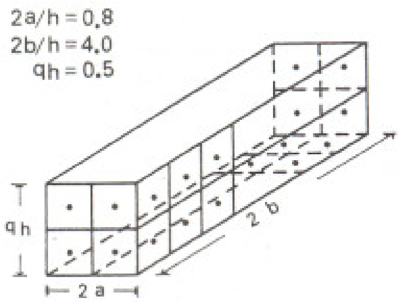

Fig. 6 Discretization schema for a box-shaped ship

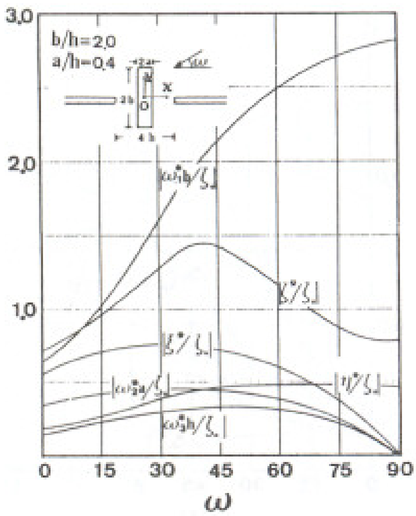

Fig. 9 Motions of a box-shaped ship ($\sigma^2 h/g$=0.75, kh=0.990, center of gravity of the ship located at $\bar{x}_0$=0, $\bar{y}_0$=0)

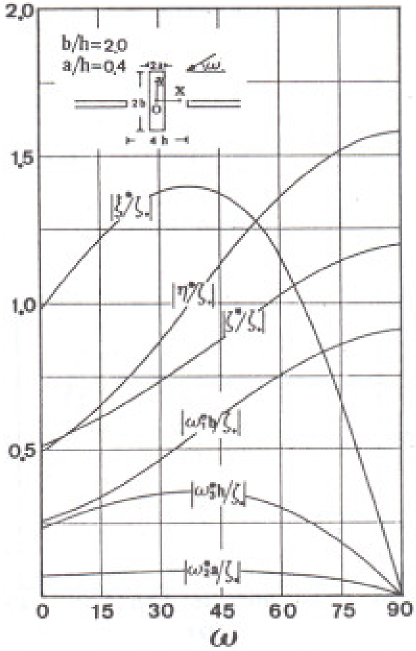

Fig. 7 Motions of a box-shaped ship ($\sigma^2 h/g$=0.25, kh=0.522, center of gravity of the ship located at $\bar{x}_0$=0, $\bar{y}_0$=0)

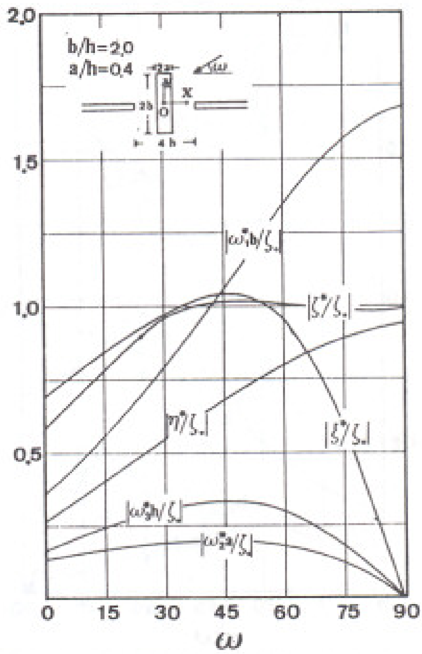

Fig. 8 Motions of a box-shaped ship ($\sigma^2 h/g$=0.5, kh=0.772, center of gravity of the ship located at $\bar{x}_0$=0, $\bar{y}_0$=0)

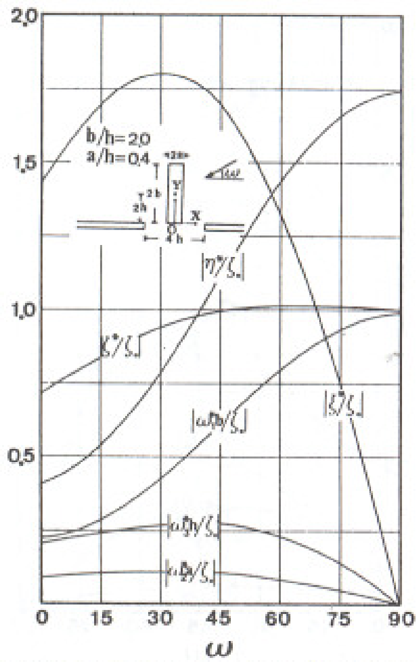

Fig. 10 Motions of a box-shaped ship (σ^2h/g=0.25, kh=0.522, center of gravity of the ship located at $\bar{x}_0$=0, $\bar{y}_0$=2h)

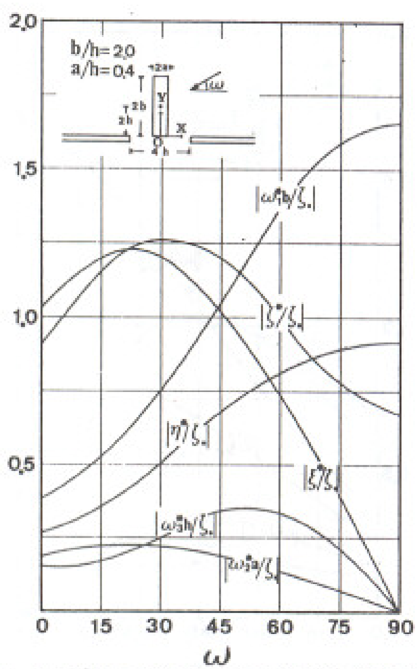

Fig. 11 Motions of a box-shaped ship (σ^2h/g=0.5, kh=0.772, center of gravity of the ship located at $\bar{x}_0$=0, $\bar{y}_0$=2h)

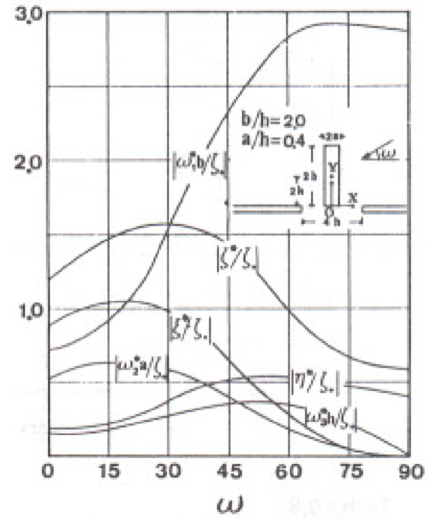

Fig. 12 Motions of a box-shaped ship (σ^2h/g=0.75, kh=0.990, center of gravity of the ship located at $\bar{x}_0$=0, $\bar{y}_0$=2h)

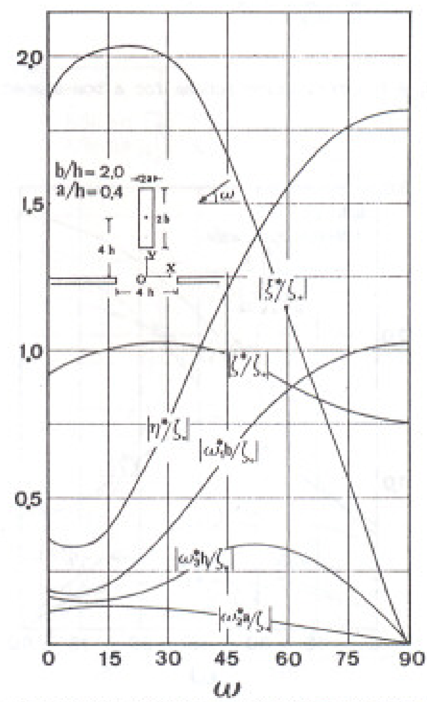

Fig. 13 Motions of a box-shaped ship (σ^2h/g=0.25, kh=0.522, center of gravity of the ship located at x_0=0, y_0=4h)

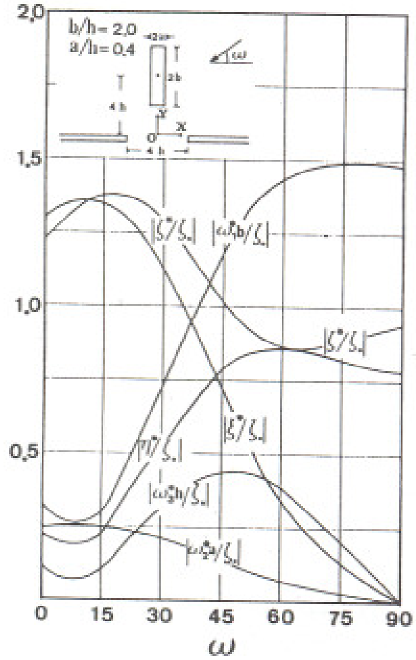

Fig. 14 Motions of a box-shaped ship (σ^2h/g=0.5, kh=0.772, center of gravity of the ship located at $\bar{x}_0$=0, $\bar{y}_0$=4h)

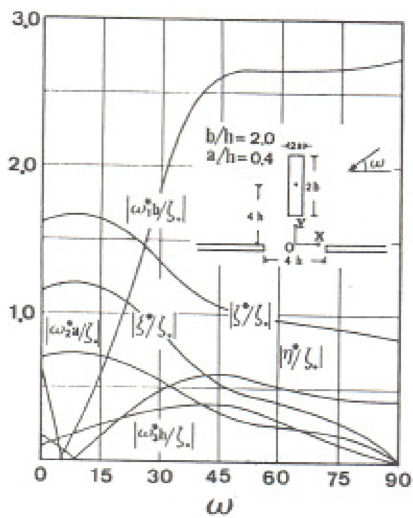

Fig. 15 Motions of a box-shaped ship (σ^2h/g=0.75, kh=0.990, center of gravity of the ship located at $\bar{x}_0$=0, $\bar{y}_0$=4h)