

邊界元素法應用於潮流解析

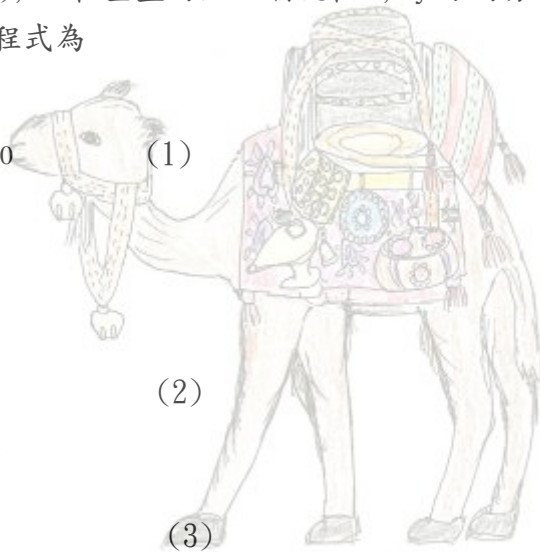
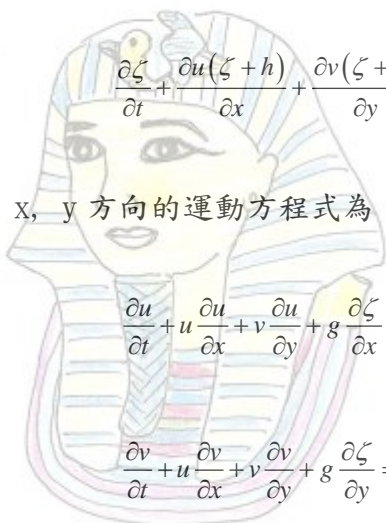
在水平面內取直角座標系(x, y), z 軸垂直向上, 潮流在 x, y 方向分量為 u, v, 水位的上昇量為 ζ 時, 連續方程式為

$$\frac{\partial \zeta}{\partial t} + u \frac{\partial (\zeta + h)}{\partial x} + v \frac{\partial (\zeta + h)}{\partial y} = 0 \quad (1)$$

x, y 方向的運動方程式為

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + g \frac{\partial \zeta}{\partial x} = 0 \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + g \frac{\partial \zeta}{\partial y} = 0 \quad (3)$$



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將(2)及(3)式的移流項忽略得

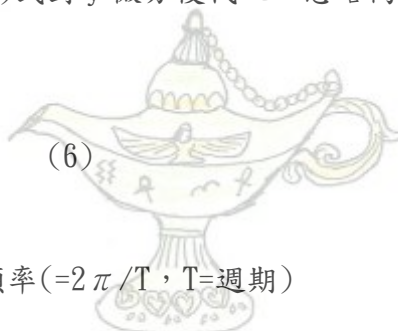
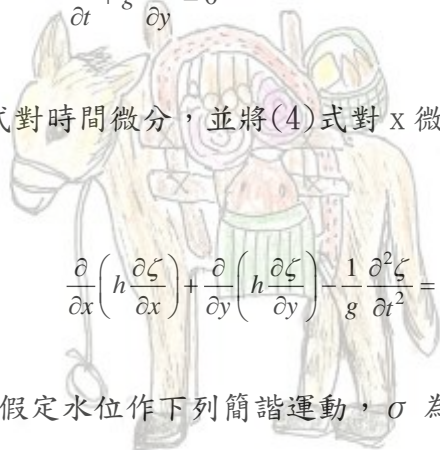
$$\frac{\partial u}{\partial t} + g \frac{\partial \zeta}{\partial x} = 0 \quad (4)$$

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$$\frac{\partial v}{\partial t} + g \frac{\partial \zeta}{\partial y} = 0 \quad (5)$$

將(1)式對時間微分, 並將(4)式對 x 微分, (5)式對 y 微分後代入, 忽略高次項得

$$\frac{\partial}{\partial x} \left(h \frac{\partial \zeta}{\partial x} \right) + \frac{\partial}{\partial y} \left(h \frac{\partial \zeta}{\partial y} \right) - \frac{1}{g} \frac{\partial^2 \zeta}{\partial t^2} = 0 \quad (6)$$



若假定水位作下列簡諧運動, σ 為角週頻率(=2 π / T, T=週期)

$$\zeta = \zeta_0 e^{-i\sigma t} \quad (7)$$

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則(6)式可改寫成

$$\frac{\partial}{\partial x} \left(h \frac{\partial \zeta}{\partial x} \right) + \frac{\partial}{\partial y} \left(h \frac{\partial \zeta}{\partial y} \right) + \frac{\sigma^2}{g} \zeta = 0 \quad (8)$$

將(7)式代入上式得

$$\nabla^2 \zeta + k^2 \zeta = -\frac{1}{h} \nabla h \cdot \nabla \zeta \quad (9)$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

將(9)式乘以加權函數 G 並積分得

$$\int G(\nabla^2 \zeta + k^2 \zeta) d\Omega = -\int G \frac{1}{h} \nabla h \cdot \nabla \zeta d\Omega$$

利用 Green 定理，上式可改寫成

$$\int \zeta(\nabla^2 G + k^2 G) d\Omega = \int G \frac{\partial \zeta}{\partial n} d\Gamma - \int \zeta \frac{\partial G}{\partial n} d\Gamma - \int G \frac{1}{h} \nabla h \cdot \nabla \zeta d\Omega$$

上式左邊，加權函數 G 能滿足下式埃及尼羅河之旅

$$\nabla^2 G + k^2 G = -\delta(Q - P)$$

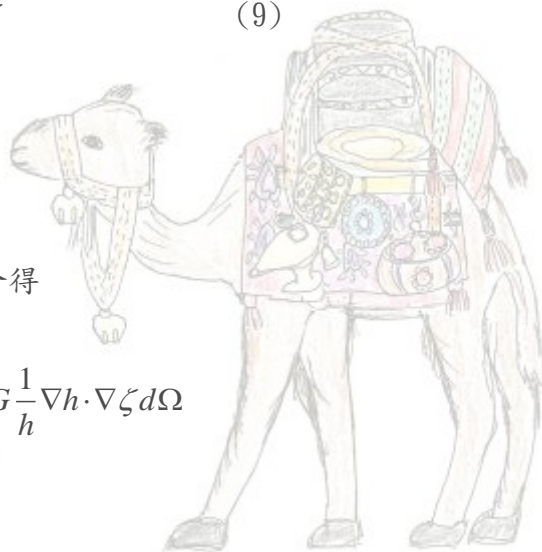
其基本解為

$$G = \frac{i}{4} H_0^{(1)}(kr)$$

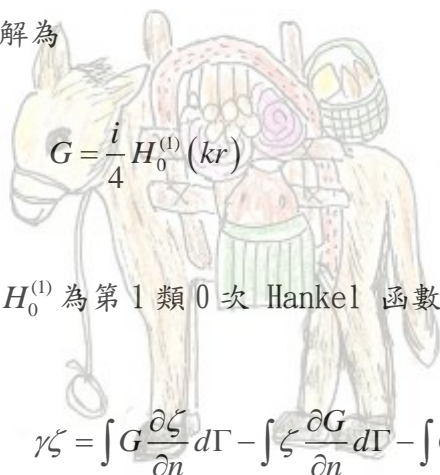
式中， $H_0^{(1)}$ 為第 1 類 0 次 Hankel 函數，故線性長波方程式的積分表示如下

$$\gamma \zeta = \int G \frac{\partial \zeta}{\partial n} d\Gamma - \int \zeta \frac{\partial G}{\partial n} d\Gamma - \int G \frac{1}{h} \nabla h \cdot \nabla \zeta d\Omega$$

在邊界上 $\gamma = 0.5$ ，在領域內 $\gamma = 1$ 。



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